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ASPECTS REGARDING THE DYNAMICS OF THE VIBRATING CONVEYORS MODELED AS 3DOF ELASTIC SYSTEMS

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ABSTRACT

The paper proposes an approach of a 3DOF (3 Degree Of Freedom) dynamic model for the vibrating conveyors, with the purpose of determining the trajectories of the points which belong to the organ to work (transport, sort out, dose, a.s.o.). This is necessary for the designing work, because it can establish some impartial criteria for the appreciation of dynamic characteristics of the equipment.
KEYWORDS: vibrating conveyor, 3DOF, elastic system, dynamics

1. INTRODUCTION

The vibrating equipment with a lot of usefulness to the transport and the sort of materials (stuffs, raw products) gross weight or by the piece, are characterized through constructive simplicity, reduced consumption of energy and precision in the technological processes (transport, dose, sort out).

Both in the designing phase and in the exploitation phase, the following performance criteria have to be imposed:

-controlled technological vibration regime;
-specific efficiency of the process;
-minimum consumption of energy;
-ordered and progressive transportation.

Figure 1 shows the constructive scheme and the operation scheme for an inertial vibratory conveyor. The conveyor can be driven also through another kind of vibrators (cinematic, electromagnetic, hydraulic, pneumatic). The parts of the conveyor in fig. 1 are the following:

1- the organ to work (eaves);
2- the inertial vibrator;
3- the two directions bearing elements (elastic and viscous damping);
4- the eccentric masses.

In general there are one, two, four or many masses in the inertial vibrators. The most useful solution is the vibrator with two eccentric masses, the same as in fig. 1; this solution is simply and it ensures a harmonical force after one direction prescription (the vibrator's axis).

In almost all experiences the organ to work is considered with the mass concentrating in the center of the mass and all forces (inertial, elastic, disturb) are passing through this point.

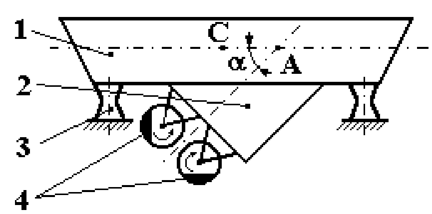

Fig. 1. Constructive scheme for an inertial vibrating conveyor

If we add at these hypotheses the consideration that the system is studied only in the longitude plane of the organ to work, it will result a lot of analytic relations which are simple and easy to use, but these relations do not explain any phenomena which can appear, the worst being the agglomeration of the material in some zones of the sieve's eaves or the conveyer's pipe.

2. PHYSICAL AND MATHEMATICAL MODELS

In this study we consider the organ to work like a solid body with viscous elastic bearings.

The elastic forces, viscous forces and disturb forces do not cross through the center mass.

The hypotheses of motion are keeping in the longitude plane of the organ to work (which is considered plane of symmetry) and we can say that the solid body has a parallel-plane motion.

In this case, the model for calculation of the vibrating equipment is shown in fig. 2.

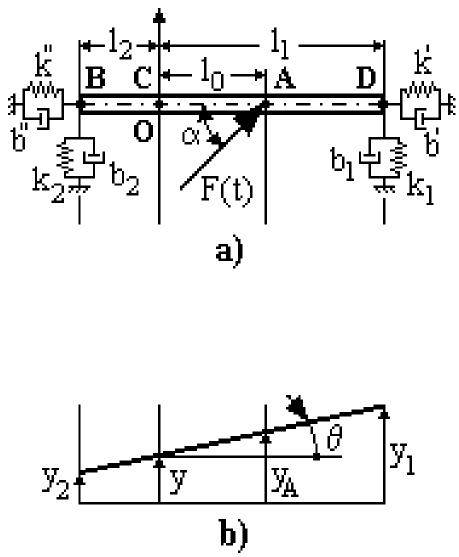

Fig. 2. Simplified calculus model of the vibrating conveyor

We consider that the solid body (the organ to work) has the **m** mass and the inertial moment $\mathbf{J_o}$ on a perpendicular axis on the longitude plane in point **C=O**. We also know the geometrical dimensions $\mathbf{l_0}$, $\mathbf{l_1}$, $\mathbf{l_2}$, the elastic constants $\mathbf{k_1}$, $\mathbf{k_2}$, $\mathbf{k'}$, $\mathbf{k''}$ and the damping coefficients $\mathbf{b_1}$, $\mathbf{b_2}$, $\mathbf{b'}$, $\mathbf{b''}$.

If we consider the horizontal position of the solid body as the static equilibrium position, the motions of the solid body can be considered the same as in fig. 2 b).

We used following hypotheses:

-the horizontal displacements and velocities of all points of the solid body are the same;

-the solid body oscillates in the longitude plane with small angles.

The inertial vibrator has one direction disturb force under an angle **α** by horizontal line.

The harmonical disturb force has the expression

$$F(t) = H \sin \omega t\,, \tag{1}$$

where

$H = m_0 e \omega^2$ is the amplitude of the force;

m_0 - total unbalanced mass;

e - the eccentricity of unbalanced mass;

ω - the angular speed of unbalanced masses.

The generalized coordinates which show the position of the solid body by the static equilibrium position are **x, y, θ**: **x** and **y** are the displacements of the center of gravity and **θ** is the rotation angle in the longitude plane.

The deformations of the linear viscous-elastic bearings are:

-for **B** point: **x** and $\mathbf{y_1}$;

-for **D** point: **x** and $\mathbf{y_2}$.

If we consider the hypothesis of a small rotation angle, the relations between displacements and deformations are the following:

$$\begin{cases} y_1 = y + l_1\theta \\ y_2 = y - l_2\theta \end{cases} \tag{2}$$

or

$$\begin{cases} y = \dfrac{l_2 y_1 + l_1 y_2}{l_1 + l_2} \\ \theta = \dfrac{y_1 - y_2}{l_1 + l_2} \end{cases} \tag{3}$$

The motion equations are determined from the second species of Lagrange equations:

$$\frac{d}{dt}\left(\frac{\vartheta E}{\vartheta \dot{q}_i}\right) - \frac{\vartheta E}{\vartheta q_i} = -\frac{\vartheta V}{\vartheta q_i} + Q_{iR} + Q_{iF}\,, \tag{4}$$

where:

E is the kinetic energy of the system;

V - the potential energy according to the elastic deformations of the bearings;

$\mathbf{Q_{iR}}$ - the generalized forces according to viscous forces;

$\mathbf{Q_{iF}}$ - the generalized forces according to disturb forces.

The kinetic energy of the system is:

$$E = \frac{1}{2} m\left(\dot{x}^2 + \dot{y}^2\right) + \frac{1}{2} J_o \dot{\theta}^2 \tag{5}$$

The potential energy is:

$$V = \frac{1}{2} k x^2 + \frac{1}{2} k_1 y_1^2 + \frac{1}{2} k_2 y_2^2 \tag{6}$$

where $k = k` + k``$.

The generalized disturb forces are:

$$Q_{xF} = \frac{\delta L_x}{\delta x} = h^* \sin \omega t \tag{7a}$$

$$Q_{y1F} = \frac{\delta L_{y_1}}{\delta y_1} = h_1 \sin \omega t \tag{7b}$$

$$Q_{y2F} = \frac{\delta L_{y_2}}{\delta y_2} = h_2 \sin \omega t \ , \tag{7c}$$

where:

$$h_1 = \frac{H \sin\alpha(l_o + l_2)}{l_1 + l_2}$$

$$h_2 = \frac{H \sin\alpha(l_1 - l_o)}{l_1 + l_2}$$

$$h^* = H \cos\alpha$$

Taking into consideration the relations (2) and/or (3) between displacements and deformations, we can write the kinetic energy

$$E = \frac{1}{2} m \dot{x}^2 + \frac{1}{2} m_{11} \dot{y}_1^2 + m_{12} \dot{y}_1 \cdot \dot{y}_2 + \frac{1}{2} m_{22} \dot{y}_2^2 \tag{8}$$

where we used the notations:

$$m_{11} = \frac{m l_2^2 + J_o}{(l_1 + l_2)^2}$$

$$m_{12} = \frac{m l_1 l_2 - J_o}{(l_1 + l_2)^2}$$

$$m_{22} = \frac{m l_1^2 + J_o}{(l_1 + l_2)^2}$$

The viscous forces are proportional to the velocities of the points **B** and **D**

$$\begin{cases} Q_{xR} = -b'\dot{x} - b''\dot{x} = -b\dot{x} \\ Q_{y_1R} = -b_1\dot{y}_1 \\ Q_{y_2R} = -b_2\dot{y}_2 \end{cases} , \tag{9}$$

Using the second species of the Lagrange’s equations, we obtain the motion equations:

$$m\ddot{x} + b\dot{x} + k_x x = h^* \sin \omega t \tag{10a}$$

$$m_{11}\ddot{y}_1 + m_{12}\ddot{y}_2 + b_1\dot{y}_1 + k_1 y_1 = h_1 \sin \omega t \tag{10b}$$

$$m_{12}\ddot{y}_1 + m_{22}\ddot{y}_2 + b_2\dot{y}_2 + k_2 y_2 = h_2 \sin \omega t \tag{10c}$$

3. DINAMIC PARAMETERS

For the vibrating technological equipment it is very important to establish the calculation relations between the vibration parameters and the trajectories of each point of the organ to work. Thus, in order to establish the correlation between dynamic parameters and technological parameters of the vibrating conveyor, the trajectory of different points of the organ to work can give concrete information on the quality of the technological process.

The equation (10a) is independent of the other two motion equations and can be written as follows

$$\ddot{x} + 2n\dot{x} + p_x^2 x = h \sin \omega t \ , \tag{11}$$

where:

$$2n = \frac{b}{m}; \ p_x^2 = \frac{k}{m}; h = \frac{h^*}{m} \tag{12}$$

The forced vibration is described by the particular solution of the equation (11) and it is as follows:

$$x = A \sin(\omega t - \varphi) \ , \tag{13}$$

where A is the amplitude of the forced vibration of the system on the **x** direction and φ is the angle between the disturb force and the displacement on the **x** direction.

Through the identification in the equation (11), we obtain the calculus relations for A and φ as follows:

$$\begin{cases} A = \dfrac{h}{\sqrt{\left(p_x^2 - \omega^2\right)^2 + 4n^2\omega^2}} \\ \tan\varphi = \dfrac{2n\omega}{p_x^2 - \omega^2} \end{cases} \tag{14}$$

The differential equations (10b) and (10b) are dynamically (inertially) coupled, with particular solutions such as:

$$y_i = C_{1i} \cos \omega t + C_{2i} \sin \omega t \quad i = \overline{1,2} \ , \tag{15}$$

or

$$y_i = A_i \sin(\omega t - \varphi_i) \quad i = \overline{1,2} \ , \tag{16}$$

where:

$$A_i^2 = C_{1i}^2 + C_{2i}^2 \tag{17a}$$

$$\tan\varphi_i = -\frac{C_{1i}}{C_{2i}} \tag{17b}$$

The velocities and accelerations are:

$$\dot{y}_i = -\omega C_{1i} \sin\omega t + \omega C_{2i} \cos\omega t \tag{18}$$

$$\ddot{y}_i = -\omega^2 C_{1i} \cos\omega t - \omega^2 C_{2i} \sin\omega t \tag{19}$$

If we introduce in the system **(10)** the relations **(15), (18)** and **(19),** we can obtain the algebraic linear system as follows:

$$\begin{cases} -m_{11}\omega^2 C_{11} - m_{12}\omega^2 C_{12} + b_1\omega C_{21} + k_1 C_{11} = 0 \\ -m_{11}\omega^2 C_{21} - m_{12}\omega^2 C_{22} - b_1\omega C_{11} + k_1 C_{21} = h_1 \\ -m_{12}\omega^2 C_{11} - m_{22}\omega^2 C_{12} + b_2\omega C_{21} + k_2 C_{12} = 0 \\ -m_{12}\omega^2 C_{21} - m_{22}\omega^2 C_{22} - b_2\omega C_{12} + k_2 C_{22} = h_2 \end{cases} \tag{20}$$

The system (20) has the unknown quantity C_{ij} $i, j = \overline{1,2}$. The matrix form of the system is

$$\underline{K} \cdot \underline{\underline{c}} = \underline{\underline{h}} \;, \tag{21}$$

where the square matrix $\underline{K}$ and the vectors $\underline{\underline{c}}$ and $\underline{\underline{h}}$ are as follow:

$$\underline{K} = \begin{bmatrix} -m_{11}\omega^2 + k_1 & -m_{12}\omega^2 & b_1\omega & 0 \\ -b_1\omega & 0 & -m_{11}\omega^2 + k_1 & -m_{12}\omega^2 \\ -m_{12}\omega^2 & -m_{22}\omega^2 + k_2 & 0 & b_2\omega \\ 0 & -b_2\omega & -m_{12}\omega^2 & -m_{22}\omega^2 + k_2 \end{bmatrix}$$

$$\underline{\underline{c}} = \begin{Bmatrix} C_{11} \\ C_{12} \\ C_{21} \\ C_{22} \end{Bmatrix}; h = \begin{Bmatrix} 0 \\ h_1 \\ 0 \\ h_2 \end{Bmatrix}$$

The solution of the system (20) is:

$$C_{11} = -\frac{U_1 M - P_1 L}{L^2 + M^2} \tag{22a}$$

$$C_{12} = \frac{U_2 M + P_2 M}{L^2 + M^2} \tag{22b}$$

$$C_{21} = \frac{U_1 L + P_1 M}{L^2 + M^2} \tag{22c}$$

$$C_{22} = -\frac{U_2 M - P_2 L}{L^2 + M^2} \tag{22d}$$

Using the relations (22), the amplitudes and the phases of the vertical coupled vibrations of the bearings **B** and **D** are

$$A_i = \sqrt{\frac{U_i^2 + P_i^2}{L^2 + M^2}}, \quad i = \overline{1,2} \tag{23a}$$

$$\tan\varphi_i = \frac{U_i M - P_i L}{U_i L + P_i M}, \quad i = \overline{1,2} \tag{23b}$$

where we used the following notations:

$$L = \left(m_{11}m_{22} - m_{12}^2\right)\omega^4 - \\ -\left(m_{11}k_2 + m_{22}k_1 b_1 b_2\right)\omega^2 + k_1 k_2$$

$$M = -\left(b_1 m_{22} + b_2 m_{11}\right)\omega^3 + \left(b_1 k_2 + b_2 k_1\right)\omega$$

$$U_1 = h_1\left(k_2 - m_{22}\omega^2\right) + h_2 m_{12}\omega^2$$

$$U_2 = h_1\left(k_1 - m_{11}\omega^2\right) + h_1 m_{12}\omega^2$$

$$P_1 = h_1 b_2 \omega; \quad P_2 = h_2 b_1 \omega$$

4. TRAJECTORIES OF THE POINTS OF THE SOLID BODY

All the points of the organ to work oscillate harmonically on directions **x** and **y**. In order to determine the trajectories of the points of the rigid body, we consider the vertical vibration of the **G** point, which is situated at a distance **z** from the center of gravity **C**, as in fig. 3.

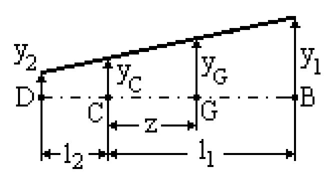

Fig. 3. Calculus model for the trajectories of the points of the conveyor eaves

From the comparison of the triangles, we can write the calculus relation of the vertical displacement of point G as follows:

$$y_G = \frac{y_2 l_1 + y_1 l_2}{l_1 + l_2} + \frac{y_1 - y_2}{l_1 + l_2} z \tag{24}$$

The trajectories of all points of the organ to work will result compounding the hortogonale vibrations done by the relations (13) and (24).

Replacing the displacements y_1 and y_1 done by relations (15) or (16) into the relation (24), the vertical vibration of the point **G** is

$$\begin{aligned} Y_G = \frac{l_2 + z}{l_1 + l_2} A_1 \sin(\omega t - \varphi_1) + \\ + \frac{l_1 - z}{l_1 + l_2} A_2 \sin(\omega t - \varphi_2) \end{aligned} \quad (25)$$

and after the calculation

$$Y_G = A_G \sin(\omega t - \varphi_G) \ , \quad (26)$$

where the amplitude and the phase are done by the following relations:

$$\begin{aligned} A_G^2 = \frac{1}{(l_1 + l_2)^2} \cdot \Big[A_1^2 (l_2 + z)^2 + A_2^2 (l_1 - z)^2 + \\ + 2 A_1 A_2 (l_2 + z)(l_1 - z) \cos(\varphi_1 - \varphi_2) \ \Big] \end{aligned} \quad (27)$$

$$\tan \varphi_G = \frac{A_1 (l_2 + z) \sin \varphi_1 + A_2 (l_1 - z) \sin \varphi_2}{A_1 (l_2 + z) \cos \varphi_1 + A_2 (l_1 - z) \cos \varphi_2} \quad (28)$$

The coordinates of the point **G** can be written

$$x_G = A \cos\varphi \sin\omega t - A \sin\varphi \cos\omega t \quad (29)$$

$$y_G = A_G \cos\varphi_G \sin\omega t - A_G \sin\varphi_G \cos\omega t \quad (30)$$

Eliminating the variable ωt from (29) and (30), it results the equation of a conic as follows:

$$\frac{x^2}{A^2} + \frac{y^2}{A_G^2} - 2 \frac{xy}{A A_G} \cos(\varphi - \varphi_G) = \sin^2(\varphi - \varphi_G) \quad (31)$$

The conic discriminant is

$$\Delta = \begin{vmatrix} \frac{1}{A^2} & -\frac{1}{A A_G} \cos(\varphi - \varphi_G) \\ -\frac{1}{A A_G} \cos(\varphi - \varphi_G) & \frac{1}{A_G^2} \end{vmatrix} \quad (32)$$

or

$$\Delta = \frac{1}{A^2 A_G^2} \sin^2(\varphi - \varphi_G) \quad (33)$$

If $\varphi - \varphi_G \neq k\pi \ (k \in Z)$, then $\Delta > 0$ and the conic is an ellipse with center in static equilibrium position of the **G** point and with **x** and $\mathbf{y_G}$ axes. The ellipse enters a rectangle which has sizes $2A$ and $2A_G$.

If $\varphi - \varphi_G = k\pi \ (k \in Z)$, then the ellipse degenerates into a straight line and the vibration of all points of the solid body are rectilinear. The inclination of the rectilinear vibration is function of the amplitudes of horizontal and vertical vibration and the phase shift as follows:

a) if $\varphi - \varphi_G = 2k\pi$, then the straight line equation is

$$\frac{x^2}{A^2} + \frac{y^2}{A_G^2} - 2 \frac{xy}{A A_G} = 0 \ \Rightarrow \ y = \frac{A_G}{A} x \quad (34a)$$

b) if $\varphi - \varphi_G = (2k+1)\pi$, then the straight line equation is

$$\frac{x^2}{A^2} + \frac{y^2}{A_G^2} + 2 \frac{xy}{A A_G} = 0 \ \Rightarrow \ y = -\frac{A_G}{A} x \quad (34b)$$

The straight line passes through static equilibrium position of **G** point and it makes with **Gx** axes the following angle:

$$\tan \theta_G = \pm \frac{A_G}{A} \quad (35)$$

If we make a rotation of the coordinate axis with θ_G angle, the coordinates of every point of the rigid body are as follows

$$\begin{cases} x = X \cos\theta_G - Y \sin\theta_G \\ y = X \sin\theta_G + Y \cos\theta_G \end{cases} , \quad (36)$$

where: (X, Y) are the coordinates of the point **G** in the rotated system with θ_G angle.

Using the linear transformation done by the relation (36) into the equation (31) of the conic, then the equation of the ellipse in the **GXY** system must be like the following conic form:

$$\frac{X^2}{S^2} + \frac{Y^2}{Q^2} = 1 \quad (37)$$

The **XY** coefficient must be zero, thus:

$$\left(A^2-A_G^2\right)sin2\theta_G-2AA_1 cos2\theta_G cos\left(\varphi-\varphi_G\right)=0 \quad (38)$$

From there it results the θ_G angle as follows:

$$tan\,2\theta_G=\frac{2AA_G\,cos\left(\varphi-\varphi_G\right)}{A^2-A_G^2} \quad (39)$$

The sizes of the semi axes of the ellipses depend on the position of the point on the solid body and are done by the following relations:

$$S=\frac{B}{\sqrt{C}} \quad (40)$$

$$Q=\frac{B}{\sqrt{D}}\;, \quad (41)$$

where:

$$B=AA_G\left|sin\left(\varphi-\varphi_G\right)\right| \quad (42a)$$

$$C=A_G^2\,cos^2\,\theta_G-\\-AA_G\,sin\,2\theta_G\,cos\left(\varphi-\varphi_G\right)+A^2\,sin^2\,\theta_G \quad (42b)$$

$$D=A_G^2\,sin^2\,\theta_G+\\+AA_G\,sin\,2\theta_G\,cos\left(\varphi-\varphi_G\right)+A^2\,cos^2\,\theta_G \quad (42c)$$

Therefore, depending on the parameters φ, φ_G, A and A_G, we conclude that the trajectory of the point **G** can be an ellipse, a circle or a straight line.

The relations which were determined can be used for any point of the organ to work.

5. CONCLUSIONS

The dynamic model for inertial vibrating conveyor presented in this study solved the following problems:

-the scheme for dynamic calculus;

-the influences of structural characteristics on the dynamic parameters of the vibrating conveyors;

-the equations of the trajectories of all points of the eaves of the vibrating conveyor in order to characterize pertinently the quality of the technological process (transportation, dosing, sort out).

With the established relations in this study, we can do the examination of the inertial vibrating conveyors or/and feeders in order to give some objective criteria of the performance level of this type of equipment.

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