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THE DYNAMIC ANALYSIS OF THE INERTIAL VIBRATING SCREENS MODELED AS 3DOF ELASTIC SYSTEMS

Assoc. Prof. Dr. Eng. Nicusor DRAGAN
MECMET - The Research Center of Machines, Mechanic and Technological Equipments
"Dunarea de Jos" University of Galati

ABSTRACT

The paper proposes an approach of a 3DOF (3 Degree Of Freedom) dynamic model for the vibrating screens, with the purpose of determining some impartial criteria for the appreciation of dynamic characteristics of the equipment. The level of specific performances of the screens is based on technological parameters of the inertial vibrating screen (frequency, disturbing force, amplitude of the screen vibration) and on the degree of uniformity of the transmission of vibration across the screen surfaces. The quality of the screening operation is much better if the vibrations are transmitted uniformly over the entire screen surface. This is why the translation vibrations must be predominant rotation vibrations, in the case of rotating inertial force. These technological requirements can be achieved after the dynamic analysis of the equipment sets up the corresponding correlation of the structural and functional parameters.
KEYWORDS: vibrating screen, 3DOF, elastic system, dynamics

1. INTRODUCTION

The inertial vibrating screens are the most used type of technological equipment for the sorting out of the construction materials (gravel, sand, crushed aggregate, limestone, a.s.o.).

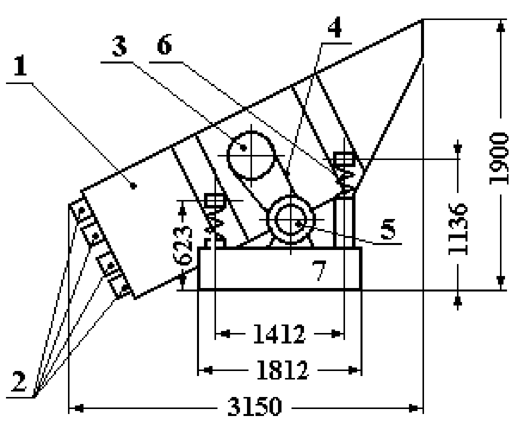

Fig. 1. Constructive scheme for the inertial vibrating screen with 5m^2 sieves

The dynamic study and the experimental determinations have been carried out for two types of inertial vibrating screens, having the screen area of 5m^2, respectively 12m^2, and the two different bearing systems: one of them with helicoidal steel springs and the other one with neoprene supports. The constructive solution of the two screens is shown in fig. 1 and fig. 2, where all overall and mounting dimensions are in mm.

Figure 1 shows the vibrating screen with four metallic sieves of 5m^2. The main parts of the equipment are:

- **1**- mainframe (welded structure);
- **2**- sieves (cutted steel plates or knitted steel wires);
- **3**- inertial vibrator (with two unbalanced masses);
- **4**- belt drive;
- **5**- electric motor;
- **6**- helicoidal steel springs;
- **7**- baseframe (welded structure).

Figure 2 shows the vibrating screen with four metallic sieves of 12m^2, where the notations are as follows:

1- mainframe (welded structure);
2- neoprene supports;
3- baseframe (welded structure);
C- the center of gravity of the mobile part of the screen;
O- the center of the rotational inertial force.

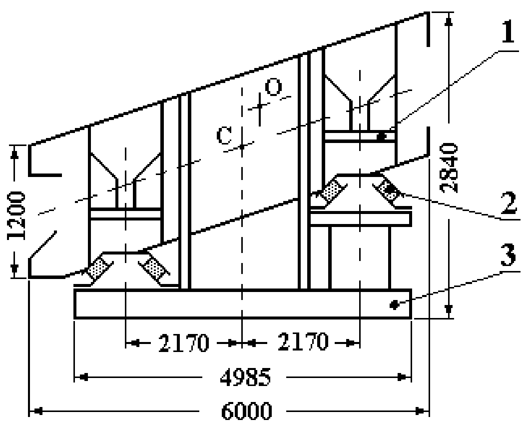

Fig. 2. Constructive scheme for the inertial vibrating screen with $12m^2$ sieves

2. PHYSICAL MODEL

The simplified calculus model of the inertial vibrating screen is shown in fig. 3. The main hypotheses of this simplified model are as follows:

- the mobile part of the vibrating screen is considered as a solid body with two planes of symmetry (vertical planes **XCZ** and **YCZ**);

- the bearings of the screen are identical and the elastic characteristics are done by the stiffness coefficients k_x, k_y and k_z;

-the perturbation rotational force is acting in the vertical plane **YCZ**; its amplitude is F_0 and the action center is O, defined by the coordinates (y_0, z_0);

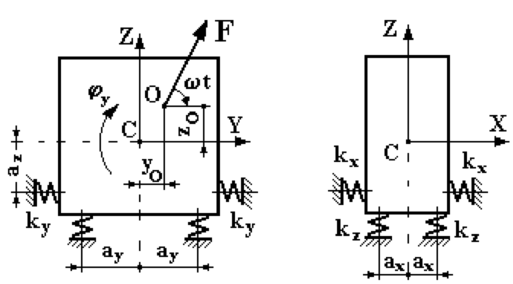

Fig. 3. Constructive scheme for the inertial vibrating screen with $12m^2$ sieves

3. MATHEMATICAL MODEL

If we consider the 6DOF model of solid body, the generalized coordinates are the natural displacements $(x, y, z, \varphi_x, \varphi_y, \varphi_z)$, where:

x - side slip motion
y - advance motion
z - lift up motion
φ_x - pitching rotation
φ_y - rolling rotation
φ_z - swing rotation

The corresponding generalized forces are as follows:

$$\begin{cases} Q_x = F_0 \cos \omega t \\ Q_y = 0 \\ Q_z = F_0 \sin \omega t \\ Q_{\varphi_x} = 0 \\ Q_{\varphi_y} = F_0 (z_0 \cos \omega t - y_0 \sin \omega t) \\ Q_{\varphi_z} = 0 \end{cases} \tag{1}$$

In order to establish the mathematical model of the solid body with elastic bearings, we consider as known:
- the dimensional and inertial characteristics (mass, central and principal inertia);
- the stiffness coefficients of the bearings;
- the disturbing force characteristics.

Considering only the elastic characteristics of the bearings, the non disturbing moving equations of the mechanical system are 2nd order differential linear equations decoupled into four subsystems as follows:

a) side slip motion and rolling rotation

$$\begin{cases} m\ddot{x} + 4k_x x + 4a_z k_x \varphi_y = 0 \\ J_y \ddot{\varphi}_y + 4a_z k_x x + 4\left(a_x^2 k_z + a_z^2 k_x\right)\varphi_y = 0 \end{cases} \tag{2}$$

b) advance motion and pitching rotation

$$\begin{cases} m\ddot{y} + 4k_y y - 4a_z k_y \varphi_x = 0 \\ J_x \ddot{\varphi}_x - 4a_z k_y y + 4\left(a_z^2 k_y + a_y^2 k_z\right)\varphi_x = 0 \end{cases} \tag{3}$$

c) the lift up motion

$$m\ddot{z} + 4k_z z = 0 \tag{4}$$

d) the swing rotation

$$J_z \ddot{\varphi}_z + 4\left(a_y^2 k_x + a_x^2 k_y\right)\varphi_z = 0 \tag{5}$$

The pulsations of the uncoupled motions for each "direction" are:

$$p_x = 2\sqrt{\frac{k_x}{m}} \tag{6a}$$

$$p_y = 2\sqrt{\frac{k_y}{m}} \tag{6b}$$

$$p_z = 2\sqrt{\frac{k_z}{m}} \tag{6c}$$

$$p_{\varphi_x} = 2\sqrt{\frac{a_x^2 k_z + a_z^2 k_y}{J_x}} \tag{6d}$$

$$p_{\varphi_y} = 2\sqrt{\frac{a_z^2 k_x + a_y^2 k_z}{J_y}} \tag{6e}$$

$$p_{\varphi_z} = 2\sqrt{\frac{a_y^2 k_y + a_x^2 k_x}{J_z}} \tag{6f}$$

The natural pulsations of the coupled motions of the four uncoupled subsystems are done by the following calculus relations:

a) side slip motion and rolling rotation

$$p_1 = \sqrt{\frac{1}{2}\left[p_x^2 + p_{\varphi_y}^2 - \sqrt{\left(p_x^2 - p_{\varphi_y}^2\right)^2 + 4\beta_1\beta_2}\right]} \tag{7}$$

$$p_2 = \sqrt{\frac{1}{2}\left[p_x^2 + p_{\varphi_y}^2 + \sqrt{\left(p_x^2 - p_{\varphi_y}^2\right)^2 + 4\beta_1\beta_2}\right]} \tag{8}$$

b) advance motion and pitching rotation

$$p_3 = \sqrt{\frac{1}{2}\left[p_y^2 + p_{\varphi_x}^2 - \sqrt{\left(p_y^2 - p_{\varphi_x}^2\right)^2 + 4\alpha_1\alpha_2}\right]} \tag{9}$$

$$p_4 = \sqrt{\frac{1}{2}\left[p_y^2 + p_{\varphi_x}^2 + \sqrt{\left(p_y^2 - p_{\varphi_x}^2\right)^2 + 4\alpha_1\alpha_2}\right]} \tag{10}$$

c) the lift up motion

$$p_5 = p_z = 2\sqrt{\frac{k_z}{m}} \tag{11}$$

d) the swing rotation

$$p_6 = p_{\varphi_z} = 2\sqrt{\frac{a_y^2 k_y + a_x^2 k_x}{J_z}} \tag{12}$$

The coupling coefficients from the relations (7)÷(10) can be calculated with the next math formulas:

$$\alpha_1 = -\frac{4}{m} a_z k_y \tag{13}$$

$$\alpha_2 = -\frac{4}{J_y} a_z k_y \tag{14}$$

$$\beta_1 = -\frac{4}{m} a_z k_x \tag{15}$$

$$\beta_2 = -\frac{4}{J_x} a_z k_x \tag{16}$$

As the motion equations are decoupled and the generalized forces are acting on three "directions" only, the differential motion equations of the forced vibration can be written as follows:

$$\begin{cases} m\ddot{y} + 4k_y y - 4a_z k_y \varphi_x = F_0 \cos\omega t \\ J_x \ddot{\varphi}_x - 4a_z k_y y + 4\left(a_z^2 k_y + a_y^2 k_z\right)\varphi_x = \\ \quad = F_0 l \sin(\omega t - \alpha) \\ m\ddot{z} + 4k_z z = F_0 \sin(\omega t - \alpha) \end{cases}, \tag{17}$$

where we used the notations:

$l = \sqrt{y_0^2 + z_0^2}$ - the distance between the gravity center **G** and the perturbation center **O**;

$\alpha = arctan\frac{z_0}{y_0}$ - the phase shift of the perturbation force ***F***.

4. AMPLITUDES OF THE FORCED VIBRATION

The particular solution of the inhomogeneous system (17) describes the forced vibration of the solid body. The amplitudes of the forced vibration are as follows:

$$A_y = \frac{F_o}{4k_z}\frac{\sqrt{P}}{R} \tag{18}$$

$$A_{\varphi_x} = \frac{F_o}{4k_z\rho_x}\frac{\sqrt{Q}}{R} \tag{19}$$

$$A_z = \frac{F_o}{4k_z\left[1 - \left(\frac{\omega}{p_z}\right)^2\right]}, \tag{20}$$

where we used the following notations:

$$P = \left[\frac{k_y}{k_z}\frac{a_z}{\rho_x}\left(\frac{a_z}{\rho_x}-\frac{z_o}{\rho_x}\right)+\left(\frac{a_y}{\rho_x}\right)^2-\left(\frac{\omega}{p_z}\right)^2\right]^2+\left[\frac{k_y}{k_z}\frac{x_o}{\rho_x}\frac{a_z}{\rho_x}\right]^2$$

$$R = \left(\frac{\omega}{p_z}\right)^4-\left[\frac{k_y}{k_z}+\frac{k_y}{k_z}\left(\frac{a_z}{\rho_x}\right)^2+\left(\frac{a_x}{\rho_x}\right)^2\right]\left(\frac{\omega}{p_z}\right)^2+\frac{k_y}{k_z}\left(\frac{a_y}{\rho_x}\right)^2$$

$$Q = \left[\frac{k_x}{k_z}\left(\frac{a_z}{\rho_x}-\frac{z_o}{\rho_x}\right)+\frac{z_o}{\rho_x}\left(\frac{\omega}{p_z}\right)^2\right]^2+\left[\frac{x_o}{\rho_x}\left(\frac{k_y}{k_z}-\frac{\omega^2}{p_z^2}\right)\right]^2$$

$\rho_x = \frac{J_x}{m}$ - the inertial radius on direction CX.

Figure 4 and fig. 5 show the amplitude characteristics for the screen with $5m^2$ sieves. Figure 6 and fig. 7 show the amplitude characteristics for the screen with $12m^2$ sieves.

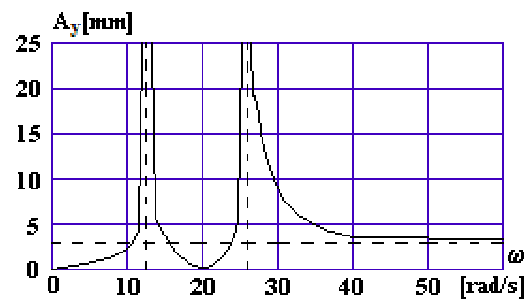

Fig. 4. The variation of the amplitude of the forced vibration on direction Y (inertial vibrating screen with $5m^2$ sieves)

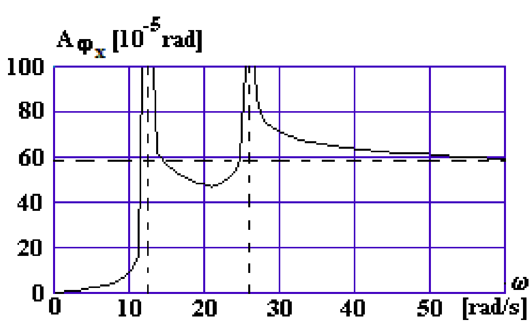

Fig. 5. The variation of the amplitude of the pitching rotation forced vibration φ_x (inertial vibrating screen with $5m^2$ sieves)

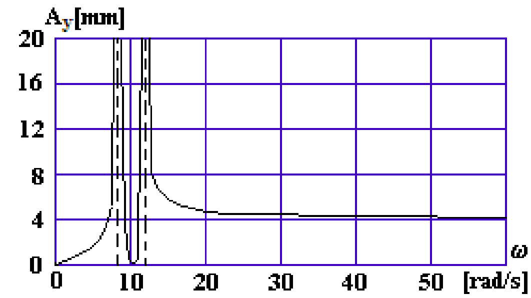

Fig. 6. The variation of the amplitude of the forced vibration on direction Y (inertial vibrating screen with $12m^2$ sieves)

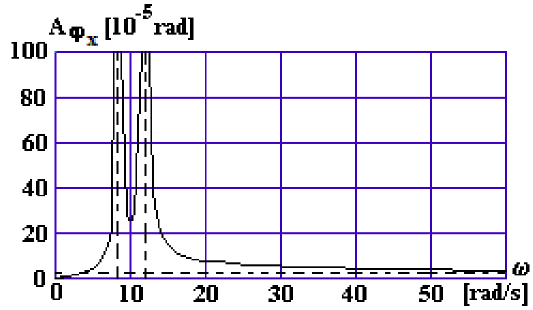

Fig. 7. The variation of the amplitude of the pitching rotation forced vibration φ_x (inertial vibrating screen with $12m^2$ sieves)

5. CONCLUSIONS

The dynamic analysis of the linear models of the vibrating screens and the plotted diagrams of the forced vibration amplitudes lead to the following conclusions:

a) the optimal technological operating condition is the post resonance vibrating regime for $\omega = (3 \div 6)p$, where $p = max(p_x, p_{\varphi_y}, p_z)$;

b) the translation vibration becomes more important than the rotational vibration when the center of mass **C** coincides with the center of perturbation force **O**.

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