THE ANNALS OF "DUNAREA DE JOS" UNIVERSITY OF GALATI
FASCICLE XIV MECHANICHAL ENGINEERING, ISSN 1224-5615
2016

DYNAMIC ANALISYS OF THE PARAMETERS OF THE MECHANICAL SYSTEMS WITH STRUCTURAL DAMPING. VISCOELASTIC SLS MODEL. PART 2: TRANSMISSIBILITY FACTOR AND ISOLATION DEGREE

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ABSTRACT

The article proposes an approach of a 1DOF (1 Degree Of Freedom) dynamic model of an elastic mechanical system with structural damping rheologically modeled as a Zener model. Zener model, also known as SLS (Standard Linear Solid) model, describes the dynamic behavior of a linear viscoelastic mechanical system under a given set of loading conditions. Rheological model is a complex parallel structure, that means: a Maxell model in parallel with a Hooke model. The system is perturbated by a harmonic force $F_0 sin\omega t$, the dynamic parameter being the amplitude of the forced steady-state vibration and the transmitted force to the base. The parametric dynamic characteristics that are drawn and analyzed are the transmissibility ratio $T(\Omega,\delta)$ and the isolation degree $I(\Omega,\delta)$.

KEYWORDS: steady-state vibration, structural damping, SLS model, transmissibility ratio, isolation degree

1. INTRODUCTION

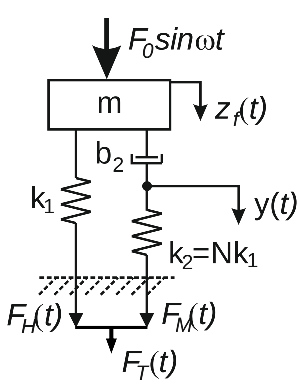

Fig. 1. Simplified scheme for 1DOF mechanical system supported by a viscoelastic Zener element

We consider the simple 1DOF mechanical system as in figure 1. If the 1DOF system is perturbated by the variable force $F(t)$, the dynamic response of the system depends on:

►the supported mass m;

►the elastic and the damping characteristics of the vertical linear viscoelastic Zener element;

►the harmonic force parameters (F_0, ω).

Considering the forced steady-state vibration of the mass m, the kinematic parameters we need for the dynamic model are: z_f , $\dot{z}_f$, $\ddot{z}_f$, y and $\dot{y}$.

The transmission paths of the dynamic force from the mass m to the base are the two rheological simple models: Hooke model and Maxwell model. We can write:

$$F_T(t) = F_H(t) + F_M(t) \quad (1)$$

2. TRANSMISSIBILITY RATIO AND ISOLATION DEGREE

Considering the linear behavior of the components of Zener model and the harmonic force $F(t)=F_0 \sin\omega t$, the moving equations and the transmitted force can be written as follows [1]:

$$\begin{cases} m\ddot{z}+b_2\left(\dot{z}_f-\dot{y}\right)+k_1\cdot z_f=F_0 \sin\omega t \\ b_2\left(\dot{z}_f-\dot{y}\right)=k_2\cdot y \\ F_T(t)=k_1\cdot z_f+k_2\cdot y \end{cases}, \quad (2)$$

where the displacements z_f, y and the transmitted force F_T are harmonic time variation with different values for the phase shift:

$$z_f(t)=A_f \sin(\omega t-\varphi_0) \quad (3)$$

$$y(t)=A_Y \sin(\omega t-\alpha) \quad (4)$$

$$F_T(t)=F_{0T} \sin(\omega t-\beta) \quad (5)$$

With the relations (3), (4) and (5), the transmitted force can be write as follows:

$$F_{0T} \sin(\omega t-\beta)= \\ =kA_f \sin(\omega t-\varphi_0)+NkA_Y \sin(\omega t-\alpha) \quad (6)$$

The amplitude F_{0T} of the transmitted force can be written [4]

$$F_{0T}=F_0\sqrt{\frac{N^2+\delta^2(N+1)^2}{N^2\left(1-\Omega^2\right)^2+\delta^2\left(N+1-\Omega^2\right)^2}}, \quad (7)$$

where:

p - natural frequency (Hooke model)

δ - structural damping ratio (Maxwell model)

Ω - relative (angular) frequency

N - elasticity coefficients ratio

Amplitude of the transmitted force can be written

$$F_{0T}=F_0\cdot T(\Omega,\delta,N), \quad (8)$$

where

$$T(\Omega,\delta,N)=\sqrt{\frac{N^2+\delta^2(N+1)^2}{N^2\left(1-\Omega^2\right)^2+\delta^2\left(N+1-\Omega^2\right)^2}} \quad (9)$$

is the transmissibility ratio.

The isolation degree is define as follows:

$$I=(1-T)\times 100 \ [\%] \quad (10)$$

3. TRANSMISSIBILITY RATIO DIAGRAMS OF 1DOF SYSTEM FORCED VIBRATION

Figures 2 to 8 show the transmissibility ratio diagrams function of the relative frequency Ω. There are considered different values for the structural damping ratio δ and elasticity coefficients ratio N.

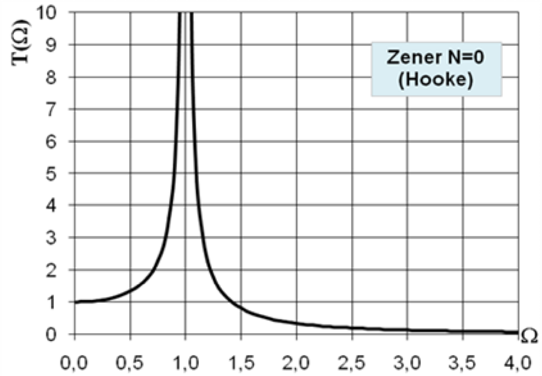

Fig. 2. Transmissibility ratio diagram Zener model - N=0 (Hooke model)

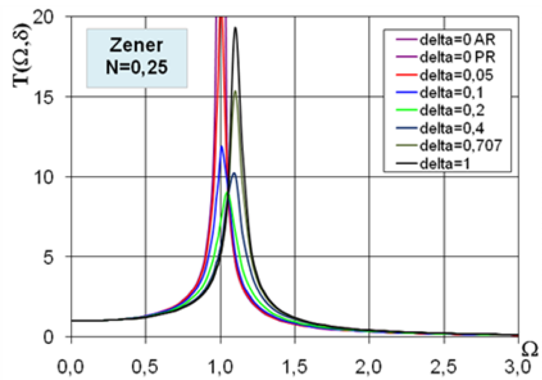

Fig. 3. Transmissibility ratio diagram Zener model - N=0.25

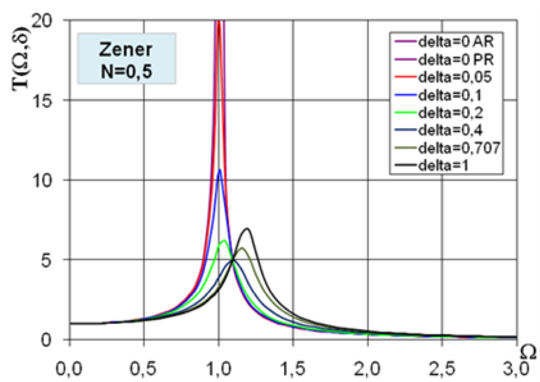

Fig. 4. Transmissibility ratio diagram Zener model - N=0.5

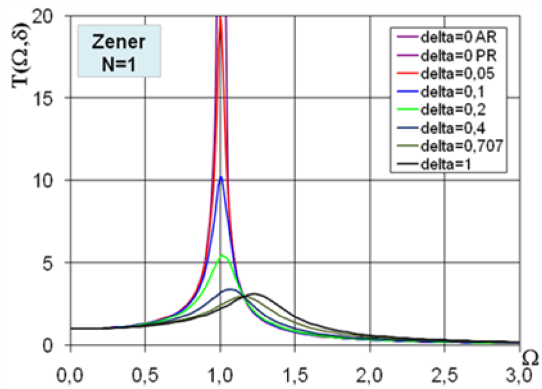

Fig. 5. Transmissibility ratio diagram
Zener model - N=1

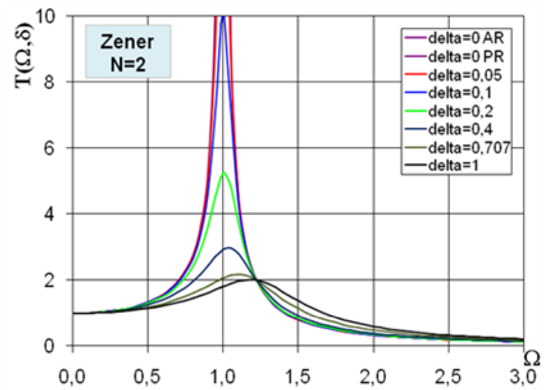

Fig. 6. Transmissibility ratio diagram
Zener model - N=2

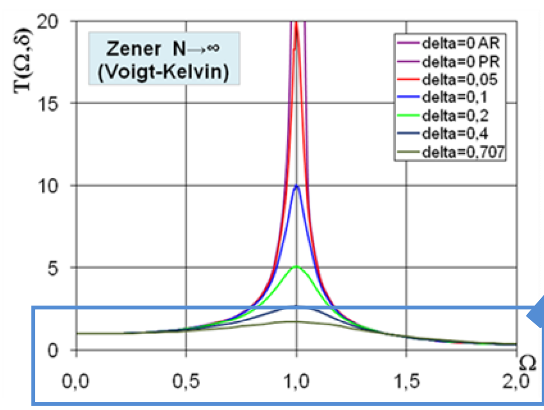

Fig. 7. Transmissibility ratio diagram
Zener model - N→∞ (Voigt-Kelvin model)

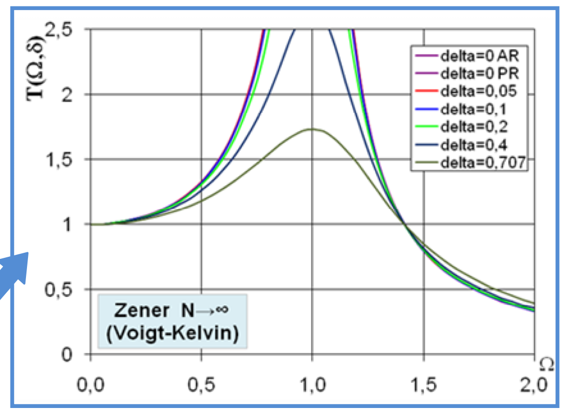

Fig. 8. Transmissibility ratio diagram detail
Zener model - N→∞ (Voigt-Kelvin model)

4. ISOLATION DEGREE DIAGRAMS OF 1DOF SYSTEM FORCED VIBRATION

Figures 9 to 14 show the isolation degree diagrams function of the relative frequency Ω. There are considered different values for the structural damping ratio δ and elasticity coefficients ratio N.

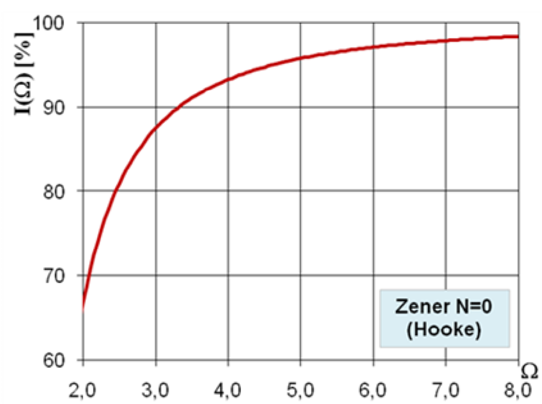

Fig. 9. Isolation degree diagram
Zener model - N=0 (Hooke model)

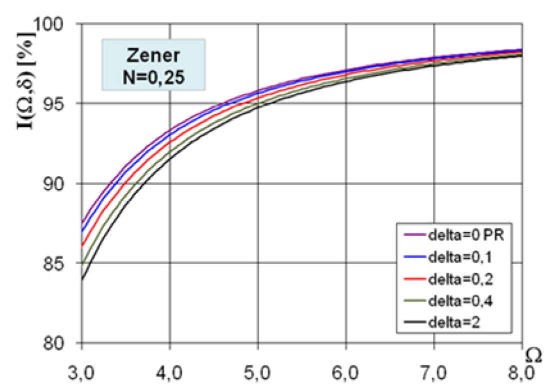

Fig. 10. Isolation degree diagram
Zener model - N=0.25

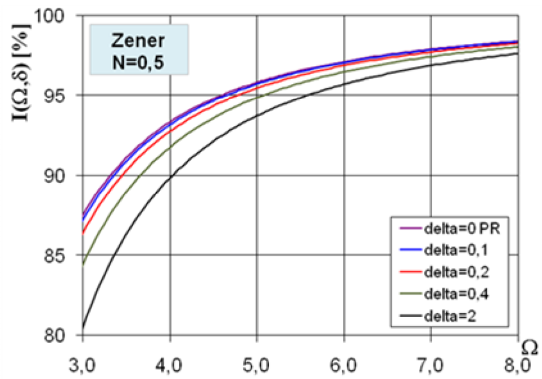

Fig. 11. Isolation degree diagram
Zener model - N=0.5

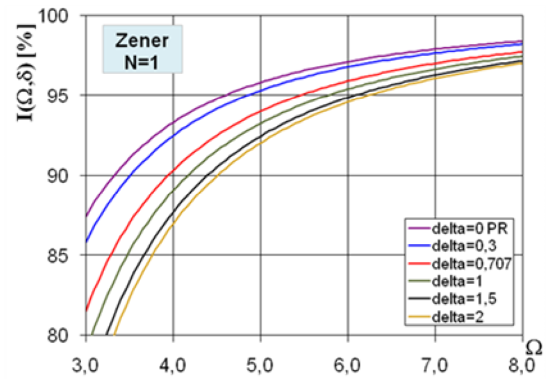

Fig. 12. Isolation degree diagram
Zener model - N=1

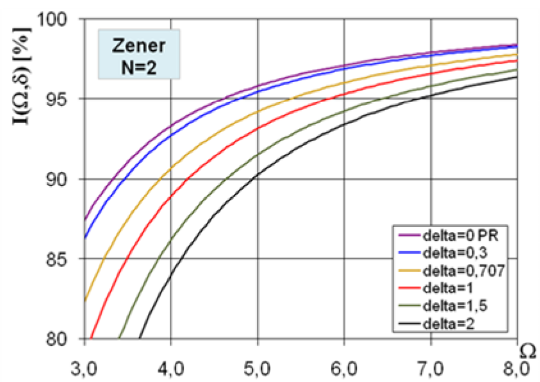

Fig. 13. Isolation degree diagram
Zener model - N=2

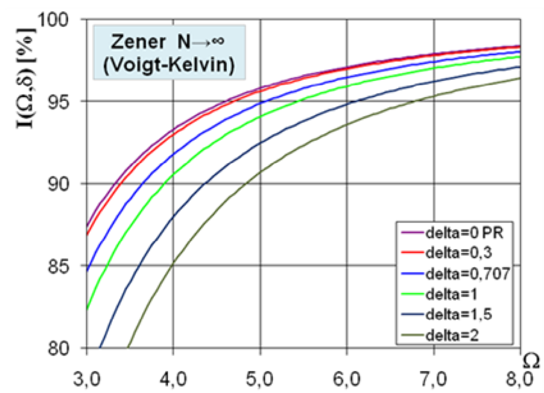

Fig. 14. Isolation degree diagram
Zener model - N→∞ (Voigt-Kelvin model)

5. CONCLUSIONS

From the transmissibility ratio parametric relation (9), we can observe that:

a)for $N=0$ or $\delta=0$ (meaning Maxwell model cancellation), Zener model becomes Hooke model; the diagram is shown in figure 2 and the transmissibility is as follows:

$$T_{N=0}(\Omega)=T_{\delta=0}(\Omega)=\frac{1}{\left|1-\Omega^2\right|} \tag{11}$$

b)for $N\to\infty$ (in Maxwell model, spring is replaced by a rigid connection, obtaining a Newton model), Zener model becomes Voigt-Kelvin model; the diagram is shown in figures 7 and 8 and the transmissibility is as follows:

$$T_{N\to\infty}(\Omega,\delta)=\sqrt{\frac{1+\delta^2}{\left(1-\Omega^2\right)^2+\delta^2}} \tag{12}$$

c)for simple rheological models (Hooke model, Voigt-Kelvin model), the maximum value for transmissibility ratio is obtained for $\Omega=1$, see figure 2 and figure 7;

d)for complex rheological Zener model, the maximum values for transmissibility ratio are obtaining for $\Omega>1$ and these value depends on the elasticity coefficients ratio N, see figures 3 to 6;

d)the isolation degree depends on the relative frequency Ω and on the dynamic structural damping δ; the smaller structural damping, the higher degree of isolation at the same relative frequency;

e)acceptable isolation degrees, $I>90\%$, can be obtained only for bigger relative frequency: Ω>3,5...4,0.for

REFERENCES

[1] **Bratu, P.,** *Elastic Systems Vibrations*, Technical Publishing House, Bucharest, 2000.

[2] **Bratu, P.,** *Bearing Elastic Systems for Machines and Equipment*, Technical Publishing House, Bucharest, 1990.