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THE CALCULATION OF PRESSURE FORCES RESULTANT ABOUT ROTOR BEARINGS OF VOLUMETRICAL PUMPS AND MOTORS WITH RADIAL PALETTES

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ABSTRACT

The objective of this paper refers to the presentation the calculation of pressure forces resultant about the palettes and the lateral rotor surface of volumic pumps and motors with radial palettes.
This calculation is necessary to give an adequate size to radial rotor bearings of those pumps and motors.

1 Geometrical and kinetic considerations

The pressure forces on radial palettes from inside the two pressure rooms (fig.1), entrance I and exit E of hydraulic medium, are each other canceling. The forces about the two palettes which separate the pressure rooms remain without balance.

$$O_2M_j = \rho_j,\ O_2M_i = \rho_i,\ O_2M_e = \rho_e$$

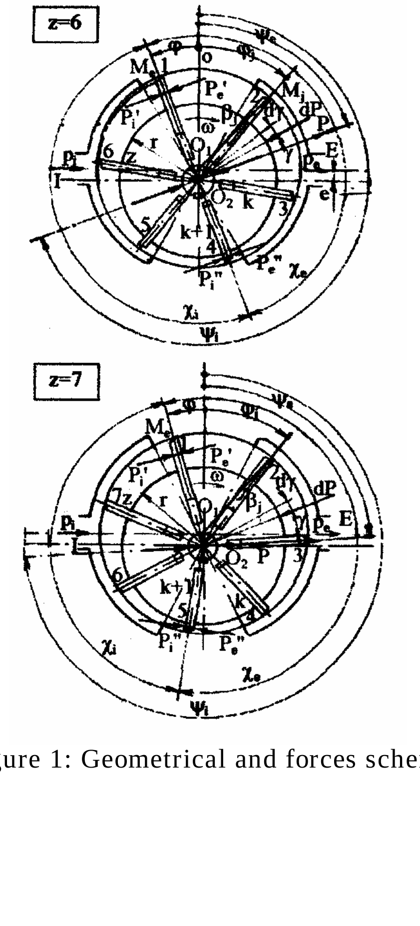

Figure 1: Geometrical and forces scheme

If the total number of volumetrical rooms (palettes) is z, of which are on dislocation in exit room is k, in entrance room is $(z-k)$ and the angular pitch of the palettes is θ, then the angles between the two radial palettes given the center of rotor O_2, which separate the pressure rooms, have the values:

$$\chi_e = k\theta,\ \ \chi_i = (z-k)\theta,\ \ \left(\theta = 2\pi z^{-1}\right), \quad (1)$$

and evidently the sum of the two angles is:

$$\chi_e + \chi_i = k\theta + (z-k)\theta = z\theta = 2\pi .$$

Also, the angles of the bisecting angles (χ_e, χ_i), given the axle of the stator centers O_1 and rotor center O_2, are:

$$\psi_e = \frac{\chi_e}{2} - \varphi = \frac{k\theta}{2} - \varphi,\ \ \psi_i = \chi_e - \varphi + \frac{\chi_i}{2} =$$
$$= k\theta - \varphi + \frac{(z-k)\theta}{2} = \frac{(z+k)\theta}{2} - \varphi, \quad (2)$$

the angle between the two bisecting angles is:

$$\psi_i - \psi_e = \chi_e - \varphi + \frac{\chi_i}{2} - \frac{\chi_e}{2} + \varphi = \frac{\chi_e + \chi_i}{2} =$$
$$= \frac{k\theta + (z-k)\theta}{2} = \pi ,$$

hence, the two bisecting angles have the same support.

At last, if φ is the position angle of the last palette (j=1) which enter in exit room, and φ_j is the angle of the current palette j from the same room, the radial movement equation of palette $[1,2]$, from the triangle $O_2O_1M_j$, can be written such as :

$$\rho_j = e\cos\varphi_j + R\cos\beta_j, \quad (e\sin\varphi_j = R\sin\beta_j). \tag{3}$$

Using the second equation, it will be result:

$$\sin\beta_j = \frac{e}{R}\sin\varphi_j,$$

$$\cos\beta_j = \sqrt{1-\frac{e^2}{R^2}\sin^2\varphi_j}:$$

$$\rho_j = e\cos\varphi_j + \sqrt{R^2-e^2\sin^2\varphi_j}.$$

For the two palettes which separate the pressure rooms, it can be written as:

$$j=1:\ \rho_e = e\cos\varphi_e + \sqrt{R^2-e^2\sin^2\varphi_e}, \tag{4}$$

$$j=k+1:\ \rho_i = e\cos\varphi_i + \sqrt{R^2-e^2\sin^2\varphi_i}.$$

Using the two palettes angles relations, it will be result:

$$\varphi_e=\varphi:\ \rho_e = e\cos\varphi + \sqrt{R^2-e^2\sin^2\varphi};$$

$$\varphi_i=\varphi+k\theta:\ \rho_i = e\cos(\varphi+k\theta) + \sqrt{R^2-e^2\sin^2(\varphi+k\theta)}. \tag{5}$$

And the sum:

$$\rho_e+\rho_i = e[\cos\varphi+\cos(\varphi+k\theta)] + \sqrt{R^2-e^2\sin^2\varphi} + \sqrt{R^2-e^2\sin^2(\varphi+k\theta)} = 2e\cos\left(\varphi+\frac{k\theta}{2}\right)\cos\frac{k\theta}{2} + \sqrt{R^2-e^2\sin^2\varphi} + \sqrt{R^2-e^2\sin^2(\varphi+k\theta)}. \tag{6}$$

2 The pressure forces resultant

The pressure forces which are on the two palettes and on the lateral surface rotor can be projected after the bisecting direction which has the position angle ψ_e, and then there sum is:

$$P = \left(P_i' - P_e' + P_i'' - P_e''\right)\cos\left(\frac{\chi_e}{2}-\frac{\pi}{2}\right) - \int_{-\frac{\chi_e}{2}}^{\frac{\chi_e}{2}} rd\gamma\, Bp_e\cos\gamma + \int_{-\frac{\chi_i}{2}}^{\frac{\chi_i}{2}} rd\gamma\, Bp_i\cos\gamma = \left(P_i' - P_e' + P_i'' - P_e''\right)\cos\left(\frac{k\theta}{2}-\frac{\pi}{2}\right) - \int_{-\frac{\chi_e}{2}}^{\frac{\chi_e}{2}} rd\gamma\, Bp_e\cos\gamma + \int_{-\frac{\chi_i}{2}}^{\frac{\chi_i}{2}} rd\gamma\, Bp_i\cos\gamma =$$

$$= \left(P_i' - P_e' + P_i'' - P_e''\right)\sin\frac{k\theta}{2} - 2rBp_e\sin\frac{k\theta}{2} + 2rBp_i\sin\frac{(z-k)\theta}{2}. \tag{7}$$

But:

$$k\theta+(z-k)\theta = 2\pi,\ \frac{k\theta}{2}+\frac{(z-k)\theta}{2}=\pi,$$

$$\frac{(z-k)\theta}{2} = \pi-\frac{k\theta}{2},\ \sin\frac{(z-k)\theta}{2} = \sin\left(\pi-\frac{k\theta}{2}\right) = \sin\frac{k\theta}{2}.$$

The resultant will be:

$$P = \left[P_i' - P_e' + P_i'' - P_e'' + 2rB(p_i-p_e)\right]\sin\frac{k\theta}{2} = [B(\rho_e-r)(p_i-p_e) + B(\rho_i-r)(p_i-p_e) + 2rB(p_i-p_e)]\sin\frac{k\theta}{2} = B(\rho_e+\rho_i)(p_i-p_e)\sin\frac{k\theta}{2},$$

$$P = B\left[2e\cos\left(\varphi+\frac{k\theta}{2}\right)\cos\frac{k\theta}{2} + \sqrt{R^2-e^2\sin^2\varphi} + \sqrt{R^2-e^2\sin^2(\varphi+k\theta)}\right](p_i-p_e)\sin\frac{k\theta}{2}. \tag{8}$$

2.1 The resultant for even number of palettes

$$\varphi\in\left[-\frac{\theta}{2},\frac{\theta}{2}\right],\ k=\frac{z}{2},\ \left(\frac{k\theta}{2}=\frac{z}{2}\frac{\pi}{z}=\frac{\pi}{2},\ k\theta=\pi\right):$$

$$P_p = B\left[2e\cos\left(\varphi+\frac{\pi}{2}\right)\cos\frac{\pi}{2} + \sqrt{R^2-e^2\sin^2\varphi} + \sqrt{R^2-e^2\sin^2(\varphi+\pi)}\right](p_i-p_e)\sin\frac{\pi}{2} = 2B\sqrt{R^2-e^2\sin^2\varphi}\,(p_i-p_e). \tag{9}$$

$$\varphi = \pm\frac{\theta}{2}:$$

$$P_{p\,min} = 2B\sqrt{R^2 - e^2\sin^2\frac{\theta}{2}}\,(p_i - p_e);$$

$$\varphi = 0:\ P_{p\,max} = 2BR(p_i - p_e).$$

2.2 The resultant for odd number of palettes

For the first half-angular pitch:

$$\varphi \in \left[-\frac{\theta}{2}, 0\right],\ k = \frac{z+1}{2},\ \left(\frac{k\theta}{2} = \left(\frac{z}{2}+\frac{1}{2}\right)\frac{\theta}{2} = \frac{z}{2}\frac{\pi}{z}+\frac{\theta}{4} = \frac{\pi}{2}+\frac{\theta}{4},\ k\theta = \pi+\frac{\theta}{2}\right):$$

$$P_i = \left[2Be\cos\left(\varphi+\frac{\pi}{2}+\frac{\theta}{4}\right)\cos\left(\frac{\pi}{2}+\frac{\theta}{4}\right) + B\sqrt{R^2 - e^2\sin^2\varphi} + B\sqrt{R^2 - e^2\sin^2\left(\varphi+\pi+\frac{\theta}{2}\right)}\right](p_i - p_e)\sin\left(\frac{\pi}{2}+\frac{\theta}{4}\right) =$$

$$= \left\{2Be\left[\cos\frac{\pi}{2}\cos\left(\varphi+\frac{\theta}{4}\right) - \sin\frac{\pi}{2}\sin\left(\varphi+\frac{\theta}{4}\right)\right]\left(\cos\frac{\pi}{2}\cos\frac{\theta}{4} - \sin\frac{\pi}{2}\sin\frac{\theta}{4}\right) + B\sqrt{R^2 - e^2\sin^2\varphi} + B\left[R^2 - e^2\left[\sin\pi\cos\left(\varphi+\frac{\theta}{2}\right) + \cos\pi\sin\left(\varphi+\frac{\theta}{2}\right)\right]^2\right]^{\frac{1}{2}}\right\}(p_i - p_e)\left(\sin\frac{\pi}{2}\cos\frac{\theta}{4} + \cos\frac{\pi}{2}\sin\frac{\theta}{4}\right),$$

$$P_i = \left[2Be\sin\left(\varphi+\frac{\theta}{4}\right)\sin\frac{\theta}{4} + B\sqrt{R^2 - e^2\sin^2\varphi} + B\sqrt{R^2 - e^2\sin^2\left(\varphi+\frac{\theta}{2}\right)}\right](p_i - p_e)\cos\frac{\theta}{4}. \quad (10)$$

$$\varphi = -\frac{\theta}{2}:\ P_i = \left[2Be\sin\left(-\frac{\theta}{2}+\frac{\theta}{4}\right)\sin\frac{\theta}{4} + B\sqrt{R^2 - e^2\sin^2\left(-\frac{\theta}{2}\right)} + B\sqrt{R^2 - e^2\sin^2\left(-\frac{\theta}{2}+\frac{\theta}{2}\right)}\right](p_i - p_e)\cos\frac{\theta}{4} =$$

$$= \left(-2Be\sin^2\frac{\theta}{4} + B\sqrt{R^2 - e^2\sin^2\frac{\theta}{4}} + BR\right)(p_i - p_e)\cos\frac{\theta}{4}; \quad (11)$$

$$\varphi = -\frac{\theta}{4}:\ P_i = \left[2Be\sin\left(-\frac{\theta}{4}+\frac{\theta}{4}\right)\sin\frac{\theta}{4} + B\sqrt{R^2 - e^2\sin^2\left(-\frac{\theta}{4}\right)} + B\sqrt{R^2 - e^2\sin^2\left(-\frac{\theta}{4}+\frac{\theta}{2}\right)}\right](p_i - p_e)\cos\frac{\theta}{4} = 2B\sqrt{R^2 - e^2\sin^2\frac{\theta}{4}}(p_i - p_e)\cos\frac{\theta}{4}; \quad (12)$$

$$\varphi = 0:\ P_i = \left[2Be\sin\left(0+\frac{\theta}{4}\right)\sin\frac{\theta}{4} + B\sqrt{R^2 - e^2\sin^2 0} + B\sqrt{R^2 - e^2\sin^2\left(0+\frac{\theta}{2}\right)}\right](p_i - p_e)\cos\frac{\theta}{4} =$$

$$= \left(2Be\sin^2\frac{\theta}{4} + BR + B\sqrt{R^2 - e^2\sin^2\frac{\theta}{2}}\right)(p_i - p_e)\cos\frac{\theta}{4}. \quad (13)$$

For the second half-angular pitch:

$$\varphi \in \left[0, \frac{\theta}{2}\right],\ k = \frac{z-1}{2},\ \left(\frac{k\theta}{2} = \left(\frac{z}{2}-\frac{1}{2}\right)\frac{\theta}{2} = \frac{z}{2}\frac{\pi}{z}-\frac{\theta}{4} = \frac{\pi}{2}-\frac{\theta}{4},\ k\theta = \pi-\frac{\theta}{2}\right):$$

$$P_i = \left[2Be\cos\left(\varphi+\frac{\pi}{2}-\frac{\theta}{4}\right)\cos\left(\frac{\pi}{2}-\frac{\theta}{4}\right) +$$

$$+B\sqrt{R^2-e^2\sin^2\varphi}+$$
$$+B\sqrt{R^2-e^2\sin^2\left(\varphi+\pi-\frac{\theta}{2}\right)}\Bigg](p_i-$$
$$-p_e)\sin\left(\frac{\pi}{2}-\frac{\theta}{4}\right)=$$
$$=\left\{2Be\left[\cos\frac{\pi}{2}\cos\left(\varphi-\frac{\theta}{4}\right)-\right.\right.$$
$$-\sin\frac{\pi}{2}\sin\left(\varphi-\frac{\theta}{4}\right)\Bigg]\left(\cos\frac{\pi}{2}\cos\frac{\theta}{4}+\right.$$
$$\left.+\sin\frac{\pi}{2}\sin\frac{\theta}{4}\right)+B\sqrt{R^2-e^2\sin^2\varphi}+$$
$$+B\left[R^2-e^2\left[\sin\pi\cos\left(\varphi-\frac{\theta}{2}\right)+\right.\right.$$
$$\left.\left.\left.+\cos\pi\sin\left(\varphi-\frac{\theta}{2}\right)\right]^2\right]^{\frac{1}{2}}\right\}(p_i-$$
$$-p_e)\left(\sin\frac{\pi}{2}\cos\frac{\theta}{4}-\cos\frac{\pi}{2}\sin\frac{\theta}{4}\right),$$
$$P_i=\left[-2Be\sin\left(\varphi-\frac{\theta}{4}\right)\sin\frac{\theta}{4}+\right.$$
$$+B\sqrt{R^2-e^2\sin^2\varphi}+ \qquad (14)$$
$$\left.+B\sqrt{R^2-e^2\sin^2\left(\varphi-\frac{\theta}{2}\right)}\right](p_i-p_e)\cos\frac{\theta}{4}.$$
$$\varphi=0:\ P_i=\left[-2Be\sin\left(0-\frac{\theta}{4}\right)\sin\frac{\theta}{4}+\right.$$
$$+B\sqrt{R^2-e^2\sin^2 0}+$$
$$\left.+B\sqrt{R^2-e^2\sin^2\left(0-\frac{\theta}{2}\right)}\right](p_i-p_e)\cos\frac{\theta}{4}=$$
$$=\left(2Be\sin^2\frac{\theta}{4}+BR+\right.$$
$$\left.+B\sqrt{R^2-e^2\sin^2\frac{\theta}{2}}\right)(p_i-p_e)\cos\frac{\theta}{4};\qquad(15)$$
$$\varphi=\frac{\theta}{4}:\ P_i=\left[-2Be\sin\left(\frac{\theta}{4}-\frac{\theta}{4}\right)\sin\frac{\theta}{4}+\right.$$
$$+B\sqrt{R^2-e^2\sin^2\frac{\theta}{4}}+$$
$$\left.+B\sqrt{R^2-e^2\sin^2\left(\frac{\theta}{4}-\frac{\theta}{2}\right)}\right](p_i-p_e)\cos\frac{\theta}{4}=$$
$$=2B\sqrt{R^2-e^2\sin^2\frac{\theta}{4}}(p_i-p_e)\cos\frac{\theta}{4};\qquad(16)$$
$$\varphi=\frac{\theta}{2}:\ P_i=\left[-2Be\sin\left(\frac{\theta}{2}-\frac{\theta}{4}\right)\sin\frac{\theta}{4}+\right.$$
$$+B\sqrt{R^2-e^2\sin^2\frac{\theta}{2}}+$$
$$\left.+B\sqrt{R^2-e^2\sin^2\left(\frac{\theta}{2}-\frac{\theta}{2}\right)}\right](p_i-p_e)\cos\frac{\theta}{4}=$$
$$=\left(-2Be\sin^2\frac{\theta}{4}+B\sqrt{R^2-e^2\sin^2\frac{\theta}{4}}+\right.$$
$$+BR\Big)(p_i-p_e)\cos\frac{\theta}{4}.\qquad(17)$$

3 Conclusions

It can be seen that the pressure forces resultant about the rotor, on an angular pitch, has a period, on even number of palettes, and it has two equal periods, on odd number.

References

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