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THE ANALYSIS OF THE DYNAMIC RESPONSE OF THE HYDROSTATIC DRIVING SYSTEMS IN A CLOSED CIRCUIT

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ABSTRACT

The hydrostatic actions in a closed circuit were remarked in the action of the technological equipment both for achievement of translation motions with linear hydrostatic engines with bilateral stem and for rotational motions with rotative hydrostatic engines. The advantages of a closed circuit result from the compactness of the whole installation, the efficient control of the mouvements at the starting and braking of the working element, the efficient control of the movement velocities of the working element and directly, through the adjustment of the cylinder volume and the smaller power consumptions then the open circuit. This paper present the results of a dynamic analysis for closed circuit hydraulic driving systems. The analysis was maded for two types of actions: with rotative, respectively, with linear hydraulic motor. The authors present the diagrams obtained for a few operating regimes.

1. Introduction

The hydrostatic actions in a closed circuit were remarked in the action of the technological equipment both for achievement of translation motions with linear hydrostatic engines with bilateral stem and for rotational motions with rotative hydrostatic engines.

The advantages of a closed circuit result from the compactness of the whole installation, the efficient control of the mouvements at the starting and braking of the working element, the efficient control of the movement velocities of the working element and directly, through the adjustment of the cylinder volume and the smaller power consumptions then the open circuit.

There are only a few advantages of this action system considered the most importants, advantages that are going to the achievement of hydrostatic components (pumps, engines) specialized to work in a closed circuit, components which contains in their structure the auxiliary elements necessary for the operating of the system (the protective valve at overcharge, cavitation and shock, the filters, the specialized adjusters in the adjustment of the cylinder volume).

The schematic diagram of the action is presented in the Figure 1.

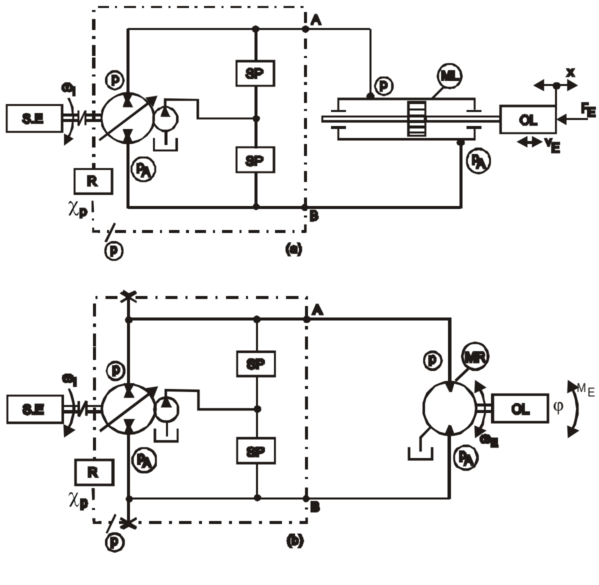

Figure 1. The schematic diagrams for hydraulic driving systems in closed circuit.

In figure 1 the first diagram is for the driving system with linear hydraulic motor, and the second is for the driving system with rotative motor. The symbols have the next signification: SE = the energy source, A = the adjuster of the pump, P = the pump, LE = the linear engine (cylinder), RE = the rotative engine, WE = the working element.

To refer to previous affirmations, these action systems are more and more used in the action of the technological equipments from diversis areas, especially where is necessary to have a permanent variation of the velocity of the working element, proportionally with the operator’s control of the equipment (the movement systems of the mobile equipments, the excavators, the cranes, the agricultural combines, the loaders, the compactors, the tractors, the rotating platforms from the crane navvy and from the mobile cranes, the action of the winches and capstans from the elevating plants and ships).

The extension of the application at the hydrostatic actions in a closed circuit have imposed researches about these applications in order to improve the operating, of the efficiency of power development and to improve the dynamic performances (the performances in permanent and non-permanent regimes).

Because the mathematical modeling of these processes of the system is very compicated, it is better to adopt a model analysis with different hypothesis of calculus, taking into consideration certain process simplifications or neglecting certain terms which have a small influence about the system, with a view to mathematical expressions which describes the system linearization.

This study analyze the action in a closed circuit in the following conditions:

- the working element is directly actuated by the hydrostatic engine of the system;
- the moment at the hydrostatic engine is constant;
- the subsystem of the thermic engine-pump is considered to be with a constant rotational speed.

It is neglected the loading slide on the specific feature of the adjuster of the thermic engine.

There are also neglecting the looses of pressure in the system’s hydraulic network, taking into consideration that are smaller then the effective value o the pressure of the system.

2. The Model of the Permanent Regime

In these specifyed hypothesis, the equations that define the system as a working one in a permanent regime are:

- for the actions of the linear engine:

$$v_E = \eta_{VP}\eta_{VM} \aleph_P \frac{V_{OP}}{2\pi A}\omega_I = \eta_{VP}\eta_{VM} v_E^* \qquad (1)$$

$$p = p_A + \frac{F_E}{\eta_{mhM} A} \qquad (2)$$

$$M_I = \frac{1}{\eta_{mhP}\eta_{mhM}} \aleph_P \frac{V_{OP}}{2\pi A} F_E \qquad (3)$$

- for the actions of the rotative engine:

$$\omega_E = \eta_{VP}\eta_{VM} \aleph_P \frac{V_{OP}}{V_{OM}}\omega_I = \eta_{VP}\eta_{VM} \omega_E^* \qquad (1')$$

$$p = p_A + \frac{2\pi M_E}{\eta_{mhM} V_{OM}} \qquad (2')$$

$$M_I = \frac{1}{\eta_{mhP}\eta_{mhM}} \aleph_P \frac{V_{OP}}{V_{OM}} M_E \qquad (3')$$

where: v_E is the mouvement velocity of the stem(for the working element); ω_E is the rotational speed at the axle of the hydrostatic engine (for the working element); η_{VP},η_{VM} is the volumetric efficiency of the pump respectively of the hydrostatic engine; $\aleph_P$ is the adjustment factor of the cylinder volume of the pump; V_{OM},V_{OP} is the maximum cylinder volume of the pump, respectively of the hydrostatic engine; A is the active area of the piston of the linear engine; ω_I is the rotational speed at the axle of the pump; p is the pressure in the active circuit of the system(A or B); p_A is the pressure in the inactive circuit of the system(B or A); F_E, M_E is the resistance force respectively the resistance moment, applied to the working element by the external medium; η_{mhP},η_{mhM} is the hydraulic-mechanical efficiency of the pump respectively of the engine; M_I is the necessary moment at the axle of the pump; v_E^* is the theoretical linear velocity; ω_E^* is the theoretical rotational speed;

The formulas (1, 1’), (2, 2’), (3, 3’) ensure the dimensioning of the action system, depending on the technological necessities of the working element ($F_E, v_E, M_E, \omega_E, \aleph_P$).

3. The Model of the Non-Permanent Regime

The model of the non-permanent regime results from the application of the continuity principle of the flow of the hydraulic agent between the pump and the engine of the action and the equation of fictitious equilibrium of forces and moments applied to working element, whence results, in the specifyed hypothesis, the following equations:

- for the actions of the linear engine:

$$\aleph_P \frac{V_{OP}}{2\pi}\omega_I = A\dot{x} + \alpha_{ML} p + \beta_{ML}\dot{p} \qquad (4)$$

$$M\ddot{x} + \gamma_{ML}\dot{x} + F_E = Ap - Ap_A \qquad (5)$$

- for the actions of the rotative engine:

$$\aleph_P \frac{V_{OP}}{2\pi}\omega_I = \frac{V_{OM}}{2\pi}\dot{\varphi} + \alpha_{MR} p + \beta_{MR}\dot{p} \qquad (4')$$

$$J\ddot{\varphi} + \gamma_{MR}\dot{\varphi} + M_E = \frac{V_{OM}}{2\pi}(p - p_A) \qquad (5')$$

where: α_{ML},α_{MR} is the coefficients of the volumetric losses of the hydraulic agent from the system, proportional with the pressure; β_{ML},β_{MR} is the coefficients that define the hydraulic capacity of the circuit; M, J is the inertial specific features of the working element; M is the reduced mass of the mouvable elements from the stem of the linear engine; J is the mechanical momentum of all the mouvable elements reduced at the axle of the rotative engine; γ_{ML},γ_{MR} is the factors of viscous resistance (Newton's type) that characterize the system.

The coefficients α and γ characterize the volumetric efficiency respectively the hydraulic-mechanical efficiency of the components of the system and they have the following expressions:

$$\alpha_{ML} = \frac{\pi}{96\eta}\left(\frac{d_M j_m^3}{b_m} + z\frac{d_P j_p^3}{b_p}\right)$$
$$\alpha_{MR} = \frac{\pi}{96\eta}\left(\frac{z_m d_M j_m^3}{b_m} + \frac{z_p d_p j_p^3}{b_p}\right) \qquad (6)$$

$$\gamma_{ML} = \frac{2\pi d_M b_m \eta}{j_m}$$
$$\gamma_{MR} = \frac{4\eta}{\pi^2}\left(\frac{V_{OM} b_m r_M}{j_m d_M} + \frac{V_{OP} b_p r_P}{j_p d_P}\right) \qquad (7)$$

The β-coefficients characterize the resilience of the action system and they have the following expressions:

$$\beta_{ML} = \frac{2V_{OR} + V_{ML}}{2E} = \frac{C_h}{2}$$
$$\beta_{MR} = \frac{2V_{OR} + V_{OM}}{2E} = \frac{C_h}{2} \qquad (8)$$

where: η is the dynamic viscosity of the hydraulic agent; E is the resilience modulus of the hydraulic agent; d_M, d_P is the diameters of the cylinder bore of the pistons, the engine or the pump; b_m, b_p is the length of the tightness threshold (M,P); j_m, j_p is the diametrically games between the pistons and the cylinder bore; z_m, z_p is the number of pistons of the engine or the pump; r_M, r_P is the disposing radius of the pistons(M,P); V_{OR} is the volume of the hydraulic agent content in the network of the hydraulic system; V_{ML}, V_{OM} is the volume of the hydraulic agent content in the linear engine or the rotative engine; C_h is the hydraulic capacity of the system.

From the equations (4) and (4’) we have been established the kinetic specific features of the working element defined through the expressions:

$$v_E = \dot{x} = \aleph_P \frac{V_{OP}\omega_I}{2\pi A} - \frac{\alpha_{ML}}{A} p - \frac{C_h}{2A}\dot{p}$$
$$a_E = \ddot{x} = -\frac{\alpha_{ML}}{A}\dot{p} - \frac{C_h}{2A}\ddot{p} \qquad (9)$$

$$\omega_E = \dot{\varphi} = \aleph_P \frac{V_{OP}}{V_{OM}}\omega_I - \frac{2\pi\alpha_{MR}}{V_{OM}} p - \frac{\pi C_h}{V_{OM}}\dot{p}$$
$$\varepsilon_E = \ddot{\varphi} = -\frac{2\pi\alpha_{MR}}{V_{OM}}\dot{p} - \frac{\pi C_h}{V_{OM}}\ddot{p} \qquad (9')$$

From the formulas (5) and (5’), in the hypothesis: $p_A = 0$, through the replacement of the formulas (9) and (9’) and through the arrangement of the terms after the differentials of the pressure , results:

$$\ddot{p} + \left(\frac{2\alpha_{ML}}{C_h} + \frac{2\gamma_{ML}}{M}\right)\dot{p} + \left(\frac{2A^2}{MC_h} + \frac{2\alpha_{ML}\gamma_{ML}}{MC_h}\right)p = \aleph_P \frac{\gamma_{ML}V_{OP}\omega_I}{\pi MC_h} + \frac{2A}{MC_h}F_E \qquad 10)$$

$$\ddot{p} + \left(\frac{2\alpha_{MR}}{C_h} + \frac{2\gamma_{MR}}{J}\right)\dot{p} + \left(\frac{2V_{OM}^2}{4\pi^2 JC_h} + \frac{2\alpha_{MR}\gamma_{MR}}{JC_h}\right)p = \aleph_P \frac{\gamma_{MR}V_{OP}\omega_I}{\pi JC_h} + \frac{V_{OM}}{\pi C_h J}M_E \qquad (10')$$

In the formulas (10) and (10’) the significance of the coefficients which precede the variables of the left member is the next one:

$$\frac{2}{C_h}\left(\alpha_{ML} + \frac{\gamma_{ML}C_h}{M}\right) = 2n_{ML}$$
$$\frac{2}{C_h}\left(\alpha_{MR} + \frac{\gamma_{MR}C_h}{J}\right) = 2n_{MR} \qquad (11)$$

The terms from expression (11) represent the damping coefficients of the systems for the case of the linear motor (ML) or for the case of the rotative motor (MR).

$$\frac{2A^2}{MC_h}\left(1+\frac{\alpha_{ML}\gamma_{ML}}{A^2}\right)=\omega_{ML}^2$$
$$\frac{2V_{OM}^2}{4\pi^2 JC_h}\left(1+\frac{4\pi^2\alpha_{MR}\gamma_{MR}}{V_{OM}}\right)=\omega_{MR}^2 \qquad (12)$$

The terms from expression (12) represent the eighen throb of the systems for the case of the linear engine (ML) or for the case of the rotative engine (MR), and

$$\frac{2A^2}{MC_h}\left(\aleph_P\frac{V_{OP}\omega_I\gamma_{ML}}{2\pi A^2}+\frac{F_E}{A}\right)=E_{ML}$$
$$\frac{2V_{OM}^2}{4\pi^2 JC_h}\left(\aleph_P\frac{V_{OP}}{V_{OM}}\omega_I\frac{2\pi\gamma_{MR}}{V_{OM}}+\frac{2\pi}{V_{OM}}M_E\right)=E_{MR} \qquad (13)$$

the terms of expression (13) represent the disturbances of the systems for the two analysed situations.

In the formula (12), we have analysed the paranthesis terms and we have established that the value of these terms tend to one and thus the expressions of the eighen throbs become:

$$\omega_{ML}\cong A\sqrt{\frac{2}{MC_h}}$$
$$\omega_{MR}\cong\frac{V_{OM}}{2\pi}\sqrt{\frac{2}{JC_h}} \qquad (14)$$

The damping coefficiens become:

$$2n_{ML}=\omega_{ML}^2\frac{1}{A^2}\left(\alpha_{ML}M+\gamma_{ML}C_h\right)$$
$$2n_{MR}=\omega_{MR}^2\left(\frac{2\pi}{V_{OM}}\right)^2\left(\alpha_{MR}J+\gamma_{MR}C_h\right) \qquad (15)$$

From these formulas result the expressions of the critical damping factors:

$$\xi_{ML}=\frac{\omega_{ML}}{2}\frac{1}{A^2}\left(\alpha_{ML}M+\gamma_{ML}C_h\right)$$
$$\xi_{MR}=\frac{\omega_{MR}}{2}\left(\frac{2\pi}{V_{OM}}\right)^2\left(\alpha_{MR}J+\gamma_{MR}C_h\right) \qquad (16)$$

and the disturbances of the system have the expressions:

$$E_{ML}=\omega_{ML}^2\left(v_E^*\frac{\gamma_{ML}}{A}+\frac{F_E}{A}\right)$$
$$E_{MR}=\omega_{MR}^2\left(\omega_E^*\frac{2\pi\gamma_{MR}}{V_{OM}}+\frac{2\pi M_E}{V_{OM}}\right) \qquad (17)$$

where: v_E^*, ω_E^* result from the formulas (1) and (1’).

With the notations achieved through the formulas (14), (16), (17) and from the expressions (10) and (10’) result the differential equations which describe the variation of the pressure in the analyzed action systems, which have the next aspect:

$$\ddot{p}+2\xi_{ML}\omega_{ML}\dot{p}+\omega_{ML}^2 p=\omega_{ML}^2\left(v_E^*\frac{\gamma_{ML}}{A}+\frac{F_E}{A}\right) \qquad (18)$$

$$\ddot{p}+z\xi_{MR}\omega_{MR}\dot{p}+\omega_{MR}^2 p=\omega_{MR}^2\left(\omega_E^*\frac{2\pi\gamma_{MR}}{V_{OM}}+\frac{2\pi M_E}{V_{OM}}\right) \qquad (18')$$

From the expressions (9) and (9’) result the variation equations of the velocity and acceleration at the working element:

$$\dot{x}=v_E^*-\frac{\alpha_{ML}}{A}p-\frac{C_h}{2A}\dot{p} \qquad (19)$$

$$\dot{\varphi}=\omega_E^*-\frac{2\pi\alpha_{MR}}{V_{OM}}p-\frac{\pi C_h}{V_{OM}}\dot{p} \qquad (19')$$

$$\ddot{x}=-\frac{\alpha_{ML}}{A}\dot{p}-\frac{C_h}{2A}\ddot{p} \qquad (20)$$

$$\ddot{\varphi}=-\frac{2\pi\alpha_{MR}}{V_{OM}}\dot{p}-\frac{\pi C_h}{V_{OM}}\ddot{p} \qquad (20')$$

4. The Modeling Results

For different hydraulic driving systems we consider the eighen throb - relation (14), the damping factors - relation (16), and the disturbances - relation (17), and it results the various behaviour of the system, included by the stabilised state of the system and the different types of dynamic instabilities. All of these it was presented in the next figures.

First time we present the diagrams for the rotative motor hydraulic driving system.

For the values: ξ= 0. 0111; ω = 1.3452 [s^{-1}]; disturbing factor = 21367078.06 [$N/m^2/s^2$], we obtained the diagram presented in the Figure 2 - pressure time evolution.

For the values: ξ= 0. 02067; ω = 2.5165 [s^{-1}]; disturbing factor = 74784773.17 [$N/m^2/s^2$], we obtained the diagram presented in the Figure 3 - pressure time evolution.

For the values: ξ= 0. 065; ω = 7.96 [s^{-1}]; disturbing factor = 0.827 10^9 [$N/m^2/s^2$], we obtained the diagram presented in the Figure 4 - pressure time evolution.

In the next diagrams are presented the diagrams for the hydraulic driving system with the linear motor.

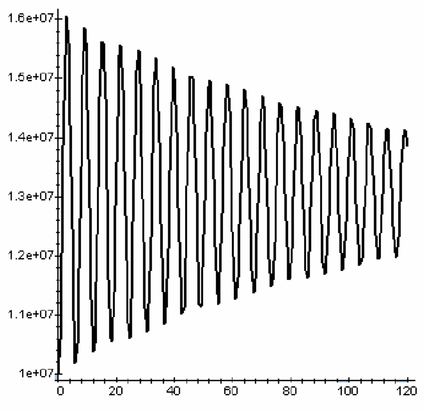

Figure 2.

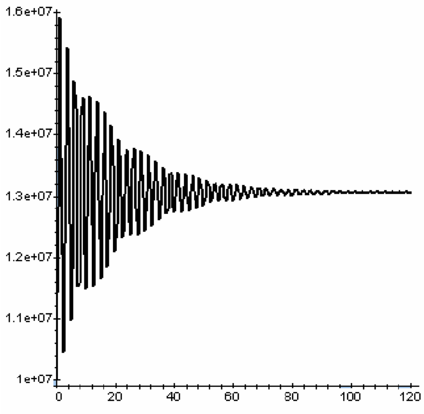

Figure 3.

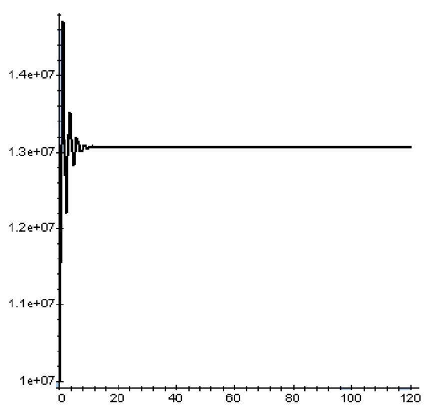

Figure 4.

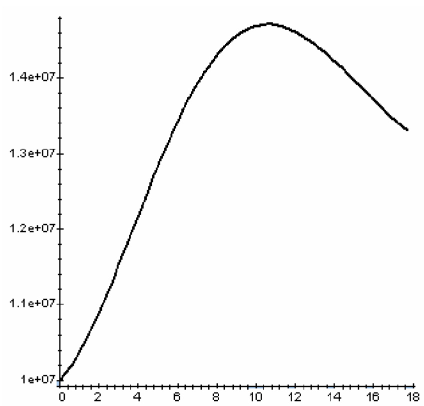

Figure 5.

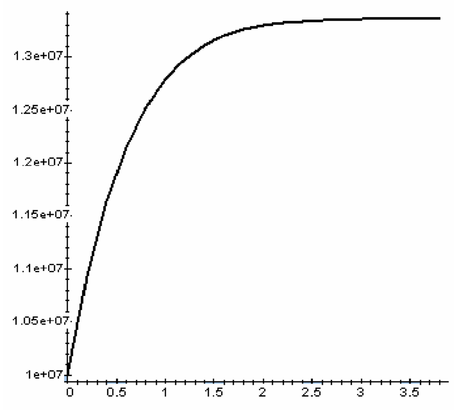

Figure 6.

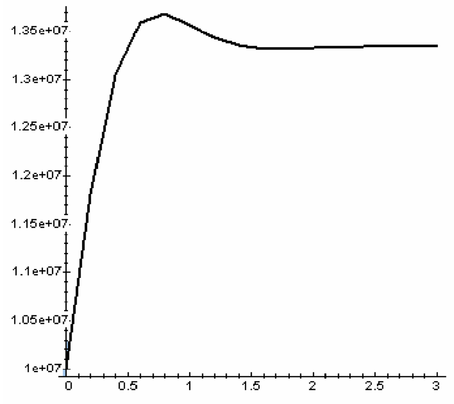

Figure 7.

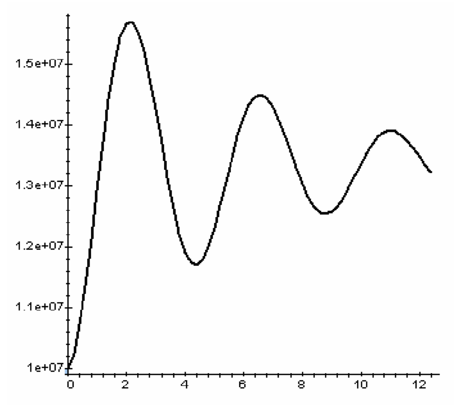

Figure 8.

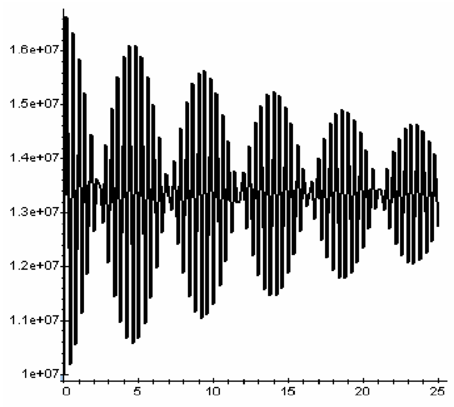

Figure 9.

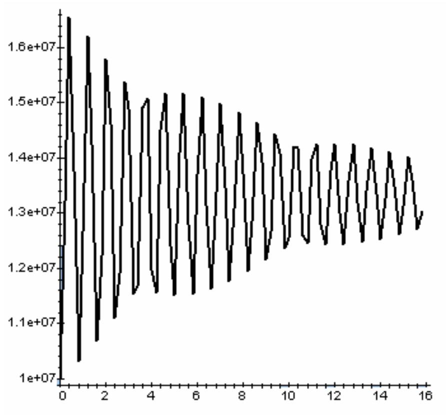

Figure 10.

For the values: ξ = 0. 00379116; ω = 21.2132 [s^{-1}]; disturbing factor = 6004896455 [$N/m^2/s^2$], we obtained the diagram of the pressure time evolution presented in the Figure 5.

For the values: ξ = 0. 017; ω = 94.87 [s^{-1}]; disturbing factor = 0.12 10^{12} [$N/m^2/s^2$], we obtained the diagram of the pressure time evolution presented in the Figure 6.

For the values: ξ = 0. 022; ω = 122.48 [s^{-1}]; disturbing factor = 0.2 10^{12} [$N/m^2/s^2$], we obtained the diagram of the pressure time evolution presented in the Figure 7.

For the values: ξ = 0. 00536; ω = 30 [s^{-1}]; disturbing factor = 0.12 10^{11} [$N/m^2/s^2$], we obtained the diagram of the pressure time evolution presented in the Figure 8.

For the values: ξ = 0. 0027; ω = 15.21 [s^{-1}]; disturbing factor = 0.3087 10^{10} [$N/m^2/s^2$], we obtained the diagram of the pressure time evolution presented in the Figure 9.

For the values: ξ = 0. 0043; ω = 23.79 [s^{-1}]; disturbing factor = 0.755 10^{10} [$N/m^2/s^2$], we obtained the diagram of the pressure time evolution presented in the Figure 10.

5. Conclusions

From the analysis maded into this paper about the dynamic behaviour of closed crcuit driving systems result that this systems have a similar behaviour with the elastic mechanical systems.

The dynamic behaviour is influenced by the eighen throb defined by the relations (14), the critical damping factors defined by the relations (16) and the disturbing factors defined by the relations (17).

On the base of this dynamic characteristics it can be evaluate the dynamic behaviour of any type of hydraulic driving system in closed circuit, like a function of mechanical variables which characterize the system (mases, moments of inertia), of hydraulic medium variables (rigidity factor, hydraulic capacity, volume, viscosity, density), and of internal constructive characteristics (diameters, areas, radius, aso)

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