THE ANNALS OF “DUNAREA DE JOS” UNIVERSITY OF GALATI
FASCICLE XIV MECHANICAL ENGINEERING, ISSN 1224 – 5615
2002

MATHEMATICAL MODEL FOR FREQUENCY-DEPENDENT SOIL PROPAGATION ANALYSIS

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ABSTRACT

Vibration analyses of advanced technology facilities typically must consider frequency as well as amplitude of vibration. A soil propagation model is proposed which will allow the use of site-specific, measurable, frequency dependent attenuation characteristics. A method is given which allows in-situ determination of those frequency-dependent properties. This approach is applied to the estimation of setback distances for various items of construction equipment at a particular site.

1. Introduction

Vibration amplitude can be quantified in terms of displacement, velocity or acceleration; each can be stated in either time or frequency domain. Issues related to vibration - sensitive facilities - such as semiconductor production plants - are generally treated in the frequency domain. Unfortunately, the literature of simplified approaches to vibration propagation in soil is very much limited to the time domain, since the areas of interest have generally been construction and building damage. These methods alone do not lend themselves particularly well to assessment in the frequency domain.

This paper presents a discussion of the frequency-dependent aspects of vibration propagation, including a propagation model which takes frequency content into consideration, and gives a method by which *in-situ* frequency - dependent propagation properties of a site can be measured and used in propagation calculations.

2. Mathematical Models of Propagation

Vibrations propagate from a source on or near the ground surface through the ground to a distant vibration-sensitive receiver predominantly by means of Rayleigh (surface) waves and secondarily by body (shear and compressional) waves. The amplitude of these waves diminishes with distance from the source. This attenuation is due to two factors: expansion of the wave front (geometrical attenuation) and dissipation of energy within the soil itself (material damping). The rate of geometrical attenuation depends upon the type of wave and the shape of the associated wave front.

Material damping is generally thought to be attributable to energy loss due to hysteresis, perhaps caused by internal sliding of soil particles.

The amount of material damping that occurs is a function of the vibration amplitude; we will limit our discussions to what are generally considered low-amplitude cases.

Material damping in soil is a function of many parameters, including soil type, moisture content and temperature. Clays tend to exhibit higher damping than do sandy soils. Wet sand attenuates less than dry sand because the pore water between sand particles carries a significant portion of compressional energy and thus does not subject compressional waves to as much attenuation by friction damping.

Propagation of Rayleigh waves is insensitive to the presence or absence of water. Frozen soil attenuates less than thawed soil.

The general equation modeling propagation of ground vibration from point “a” (a location at distance r_a from the source) to point “b” (a location at distance r_b from the source) may be stated in the form of Equation (1),

$$v_b = v_a \left(\frac{r_a}{r_b} \right)^{\gamma} e^{\alpha (r_a - r_b)} \tag{1}$$

where γ is a coefficient dependent upon the type of propagation mechanism and a is a material damping coefficient.
Theoretical radiation models based upon half-space formulation have been used to determine γ corresponding to various propagation models in idealized cases. This form of attenuation can also be expressed in terms of decibels per doubling of distance. Several commonly accepted values of γ are shown in Table 1 for a number of types of sources and waves.

Table 1

Source	Measurement point	γ
point on surface	surface	0.5
point on surface	surface	2
point at depth	surface	1
point at depth	depth	1

Most settings involve surface (or near-surface) sources and receivers, and Rayleigh wave propagation is the most common. Even when the actual vibration source is below the surface — as with pile driving — Rayleigh waves are formed within a few meters of the point on the surface directly above the source, and the propagation can be modeled in terms of Rayleigh waves. There are two principal forms in which investigators have fit Equation (1) to observed data. One approach is to neglect damping attenuation and fit geometric attenuation curves to field data.
The other approach assumes Rayleigh wave propagation and fits material damping curves to measured data. In the first approach, one sets $\alpha = 0$ and assumes that attenuation follows a straight line on a log-log plot of velocity amplitude as a function of distance. In this case, γ is the slope of that line in decades per decade. Investigators have found values of γ between 0,8 and 1,7. Table 2 summarizes some of the published values of γ.

Table 2

Investigator	Soil type	Geometric attenuation	
		γ	dB/doubling
Wiss	sands	1.0	6
	clays	1.5	9
Brenner	surface sands	1.5	9
	sand fill over soft clays	0.9	5-6
Attewell, Farmer	various soils	1.0	6
Nicholls, Johnson, Duvall	firm soils and rock	1.4-1.7	8.5-10
Martin	clay	1.4	8.5
	silt	0.8	5
Amick	clay	1.5	9

Using the other approach, one sets g = 0.5 and selects a value of a based upon soil type.

3. Frequency - Dependent Material Attenuation

Barkan and Dowding observe that a soil’s material damping provides a specific amount of attenuation per wavelength. This is consistent with observations for many propagation media. It is typical to assume a damping material property that is constant for a wide range of frequencies.
There are many numeric quantities used to denote damping, including loss factor η, damping ratio ζ, resonance amplification factor Q, or what Richart, Hall and Woods define as the “specific dissipation function” 1/Q.
When a “ringing” vibrating medium is undergoing decay due to damping, the ratio of successive cycles δ, the “log decrement,” provides another measure of material damping. These terms are all related, as defined by Equation (2).

$$\eta = 2\varsigma = \frac{\delta}{\pi} = \frac{1}{Q} \tag{2}$$

Richart, Hall and Woods define the relationship between a and these material coefficients as shown in Equation (3),

$$\delta = \frac{2\pi c\alpha}{\omega} = \lambda\alpha = \frac{c\alpha}{f} \tag{3}$$

where c is the wave velocity, λ is the wavelength, and ω is the circular frequency ($\omega = 2\pi f$). This can be restated in terms of loss factor, as in Equation (4).

$$\alpha = \frac{\eta\pi f}{c} \tag{4}$$

For a particular soil deposit, we can assume that both η and c represent constant soil properties, therefore a quantity made up of the ratio of the two, $\rho = \eta/c$, can also be considered a property of that soil.
Thus, a new relationship can be used to define a:

$$\alpha = \rho\pi f \tag{5}$$

Woods and Jedele have proposed a classification of earth materials by attenuation

coefficient. Dowding summarizes this in terms of ranges of a at 5 Hz and 50 Hz, but if one uses Equation (5) as a definition of α, one can arrive at a tabulation of ρ as a function of earth material type.
This is given in Table 3. The constant ρ can be used in propagation models of the form given in Equation (6).

$$\frac{\nu_b}{\nu_a} = \left(\frac{r_a}{r_b}\right)^{\gamma} e^{\rho\pi f\left(r_a - r_b\right)} \qquad (6)$$

Table 3

class	description	α at 5 Hz	ρ
I	weak or soft soils (soils penetrates easily): -loessy soils -mud -loose beach sand -dune sand -organic soils -topsoil	0.003-0.01	2 10^{-4} to 6 10^{-4}
II	competent soils (can dig with shovel): -most sands -sandy clays -silty clays -gravel -silts	0.001-0.003	6 10^{-5} to 2 10^{-4}
III	hard soils (must use pick to break up): -dense compacted sand -dry consolidated clay -consolidated glacial till -some rocks	0.0001-0.001	6 10^{-6} to 6 10^{-5}
IV	hard, competent rock (difficult to break with hammer): -bedrock -hard rock	<0.0001	<6 10^{-6}

The vibration attenuation (in dB) between points "a" and "b" can be stated in the form of Equation (7).

$$At = VL_b - VL_a = 20\ log\left({\nu_b}/{\nu_a} \right) \qquad (7)$$

We can rewrite Equation (1) to define attenuation of Equation (7).

$$At = A_\gamma + A_\alpha$$

where

$$A_\gamma = 20\ \gamma\ log_{10}\left(\frac{r_a}{r_b}\right) \qquad (8')$$

$$A_\alpha = 8{,}68\ \alpha\left(r_a - r_b\right) \qquad (8'')$$

both terms being expressed in decibels.

4. Procedure for Measuring Site-Specific Attenuation Properties

When α is traditionally determined from time-domain field measurements, it is generally necessary to obtain measurements of a single activity at several distances. It is difficult to obtain these data at great distance without "contamination" by ambient vibrations. However, one may obtain the frequency - independent damping characteristics (such as ρ or the loss factor η) for use in frequency - dependent analysis from a pair of spectra obtained simultaneously at two distances from a source. These distances need not be great, just enough to be assured that the wave propagation mechanism is via Rayleigh waves.
Assuming that the propagation was due to Rayleigh waves, one can determine ρ by carrying out the following steps at each frequency:

- Subtract scalar $A\gamma$ (3 decibels) —calculated using Equation (8a) for γ = 0.5 —from the transfer function, leaving spectrum $A\alpha$.
- Divide each frequency component of $A\alpha$ by $8.68(r_b - r_a)$ - from Equation (8''), obtaining α as a function of frequency.
- Compute ρ at each frequency using Equation (9), obtained by rearranging Equation (5).

$$\rho = \frac{\alpha}{\pi f} \qquad (9)$$

- There will be some variation as a function of frequency, depending upon the coherence of the transfer function. Calculate the average ρ for the frequencies for which the transfer function can be assumed statistically adequate (and discard data "contaminated" by ambient conditions).
- Calculate the theoretical site-dependent material attenuation spectrum using Equation (10).

$$A_\alpha = 8{,}68\rho\pi f\left(r_b - r_a\right) \qquad (10)$$

- Add $A\gamma$ to Aα to obtain the attenuation spectrum.

The conclusion one may draw from this form of attenuation curve is that higher-frequency vibrations are attenuated more rapidly with

distance than are low-frequency components. At distances quite close to an impact source (such as a pile driver) the peaks in the time history are quite sharp — indicative of high frequencies — but at a great distance the peaks are smooth and more undulating — showing that the high frequency components have indeed been attenuated.
If a two-channel spectrum analyzer is available, the transfer functions can be measured directly.
In addition, the analyzer can be used in time-domain mode to obtain the travel time between the two measurement locations of a pulse generated at the driving location. This can be used to calculate sitespecific Rayleigh wave velocity. It should be noted that the scraper was able to define a reasonably shaped attenuation curve because it generated a time history that was somewhat stationary and relatively free of impulses. Other sources which generated vibrations more impulsive in nature did not produce such consistent results.

5. Conclusions

Vibration analyses of advanced technology facilities typically must consider frequency as well as amplitude of vibration.
A soil propagation model has been proposed which will allow the use of sitespecific, measurable, frequency dependent attenuation characteristics. A method has been proposed which allows *in-situ* determination of those frequency-dependent properties.

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