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# PHISYCAL AND MATHEMATICAL MODEL FORVIBRATORY DRIVING PROCESS ANALYSIS

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## ABSTRACT

*Vibratory driving of pile is one of the technologies used for powerless basement soil deep consolidation. This process consist by favouring the pile penetration in soil, whereupon when vibrations using. Experimentally, was determined that the soil subjected to the harmonical movement, with certain amplitude and frequency, this behave like a viscous liquid and the penetration process of different shape bodies was favorized. The amplitude and the frequency of excitatory force have specific values for each type of soil. This paper present a numerical model for computer analysis of this process. For a better behaviour of the modeling is necessary to corelate the computer program values with those that the exepriments was proved.*

## 1. Introduction

Vibrodriving of different consolidation elements in soil is a component part of the basement ground consolidation technologies. In fact, it is a technology of deep ground consolidation, and it is based on the supporting of construction foundation on the deeply solid state ground. Practically, it will be penetrate the surface powerless ground with consolidation element, like piles, tubes, forepoles, walls, and will be make the foundation supporting on this elements system. Thus, the construction will be keep away from great settlements of surface ground.

For this process it will be necessary specifically equipments, which will driving the element into the soil. Generally, exist a few types of driving machines.

One of this are based on the statically pushing process, using a big value force, with constant modulus and direction.

Another machines type are based on the percussion force produced by a heavy mass, which is free falling from selected stroke height and which transfer a significant part of his kinetically energy, at the impact moment, to driven element. The energy for lifting the ram is produced by hydraulic device, mechanical system or Diesel engine.

In the last time, it was observed that the necessary energy for penetration the element in soil it will be diminished if the pushing force have vibratory character or if the ground is vibratory excited. Based on this observations, it was developed same driving equipment’ s which put into the practice this fact. It is form the third category of driving machines. Because the necessary driving energy is smaller the others, this are the most used equipments at this moment.

For this category of vibratory devices, and not only for them, it is interesting to analyze and optimize the supply power, taking into account all the system requests like operating parameters and final job characteristics.

With the other words, which is the minimal energy that is necessary to introduce in this system for obtained the element penetration with right parameters - this the question at which we try to answer en route this work.

For solve this job it was consider the mechanical model of the equipment and it was analyzed. Also, it was analyzed the interaction work tool – soil.

But, the results was not verified on practice, because this system is like a single equipment, which must be taking all over, with driving device, driven element and soil and taking into account the interaction between all this elements. It must be analyze consider all this components in a single physical and mathematical model.

The authors was develop a physical and a mathematical model which was combined the equipment aspects, the soil – element interaction phenomenon and the reological law to estimate soil behaviour under load.

## 2. Physical Model for Pile Vibratory Driving

It was considered a driving equipment with power supply assured from asynchronous electrical engine (AC motor). This is elastic supported on the centrifugal generator. The generator have two eccentrically masses, which have rotational antiphasic movement. This device produce unidirectional harmonically force, characterized by amplitude $F_o$ and angular velocity (frequency f).

In first approximation, the working diagram of the electrical device (the moment – revolution characteristic) was supposed to be constant, that mean the engine assure the nominal angular velocity whatever will be the resistant moment (form null to maximal value).

For consolidation element it was chosen a pile, with round cross-section.

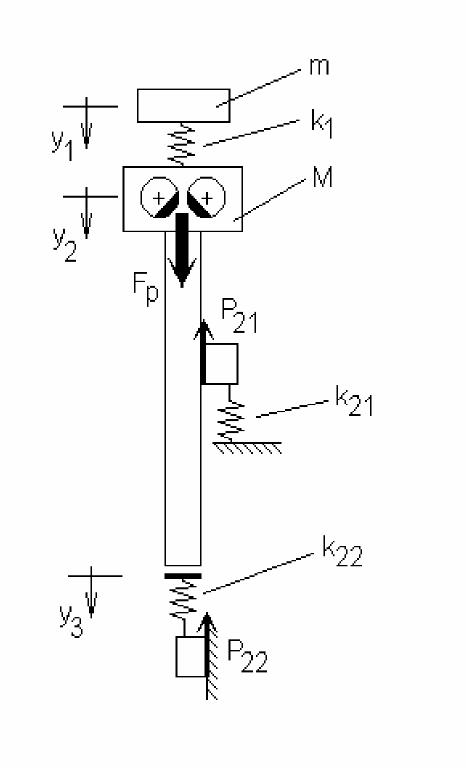


Figure 1. Physical model of vibratory driving equipment.

The physical model which is presented by authors, have diagram in Figure 1. The symbols have the next signification: M - driving device mass (without the engine and include the pile); m – engine mass; $k_1$ –rigidity constant of engine support; $k_{21}$ – soil rigidity constant on side surface of the pile; $k_{22}$ – soil rigidity constant on the top of pile; $P_{21}$ – soil plasticity limit strain (given from limit shear strain) on side surface of pile; $P_{22}$ – soil plasticity limit strain (given from limit compressive strain) on the top of pile.

## 3. Mathematical Model of the Considered System

The analysis will be make with consider the vibratory supplier working in a steadiness state. This means that the disturbing force produced by eccentrically device have the form presentes by Equation (1).

$$F_p = F_o \cos(\omega t + \varphi) \qquad (1)$$

In the previous expression we can suppose the initial phase $\varphi$ to be null.

The equations which describe the movement of this model are obtained by using the D' Alambert Principle, and these are presented by the next expressions.

$$\begin{cases} m\dfrac{d^2 y_1}{dt^2} + k_1(y_1 - y_2) = 0 \\ M\dfrac{d^2 y_2}{dt^2} - k_1(y_1 - y_2) + F_{r1} + F_{r2} = F_p \end{cases} \qquad (2)$$

In equations (2) $y_1$ and $y_2$ are the movements of engine device, respectively of generator – pile system. The two forces $F_{r1}$ and $F_{r2}$ are resistance forces produced at pile – soil interface (on side surface and at the top).

If supposed that the right reological model for this case is the Bingham model, which in this first approximation, will be considered without damping, it is possible to describe the analytical expressions for $F_{r1}$ and $F_{r2}$ forces.

On the side surface of the pile appear forces due to friction between pile and soil. By these forces, the pile movement involved the moving of soil grains. This are happened in elastically domain (therefore without soil deplacement) until the forces become equal with shear strain of soil structure, when the soil movement started. In case of cohesion's soils the limit strain increasing with the cohesion value (Coulomb's law). This stopped at the same time with the pile and restarted, on the other stroke, in the same conditions.

Based on these, the expression of side resistant force is:

$$F_{r1} = \begin{cases} k_{21} z_2 \leftarrow z_2 < P_{21}/k_{21} \\ P_{21} \dfrac{|\dot{y}_2|}{\dot{y}_2} \leftarrow z_2 \geq P_{21}/k_{21} \end{cases} \quad (3)$$

The other resistant force $F_{r2}$ can be described in the same way that $F_{r1}$, differences appears at the limit strain when soil movement started. In this case we supposed that the axial limit pressure is the value that decide if soil movement started or not. Now, the soil deplacement matter because is the value of pile penetration.

Therefore, the top resistant force is:

$$F_{r2} = \begin{bmatrix} penetration: \begin{cases} k_{22} z_2 \leftarrow z_2 < P_{22}/k_{22} \\ P_{22} \dfrac{|\dot{y}_2|}{\dot{y}_2} \leftarrow z_2 \geq P_{22}/k_{22} \end{cases} \\ lifting: \begin{cases} k_{22} z_2 \leftarrow z_2 < P_{22}/k_{22} \\ 0 \leftarrow z_2 \geq P_{22}/k_{22} \end{cases} \end{bmatrix} \quad (4)$$

From equation (4) result that the $F_{r2}$ expression have two separately forms for every strokes: one for penetration (descend stroke) and one for lifting. The difference appear after the limit force $P_{22}$ was exceeded: on descending stroke appear the pile penetration and on lifting stroke the pile break off with the soil (at the top).

Considering all the equations written until here we can solve this mathematical model. The authors was adopted the 4th order Runge Kutta method and was written a dedicated computer software to solve this problem.

## 4. Numerical Analysis

Solving equations (1) to (4) with null initial conditions for all parameters, we obtained the pile movement time function, the time evolution of total resistant force and, of course, the penetration. Figures 2 and 3 present this numerical results.

Analysing the Figure 2, the first segment of time (see the second data set) correspond to the transitory state period of movement. For $F_{r1}$ = 30 N, $F_{r2}$ = 50 N and amplitude $F_o$ of excitation are 12 kN, it easy to observe that transiotry time is less that one second. After that, the movement was stabilized.

This fact also conduct at the conclusion that the available power in the system is enough, is adequate with the vibratory driving process requests.

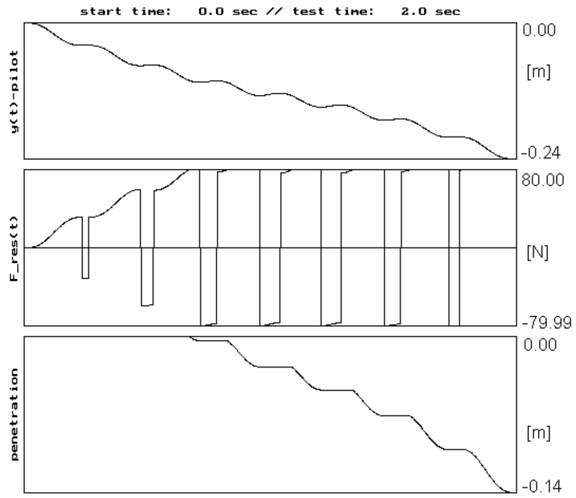


Figure 2. Diagrams for pile deplacement, penetration and total resistant force, for initial transitory state.

Also, in this diagram appear the two zones of pile movement:

- elasticity zone – when the penetration is null and resistance force are diminished, and
- plasticity zone – when penetration is equal with pile deplacement and resistance force grow up to the maximum value.

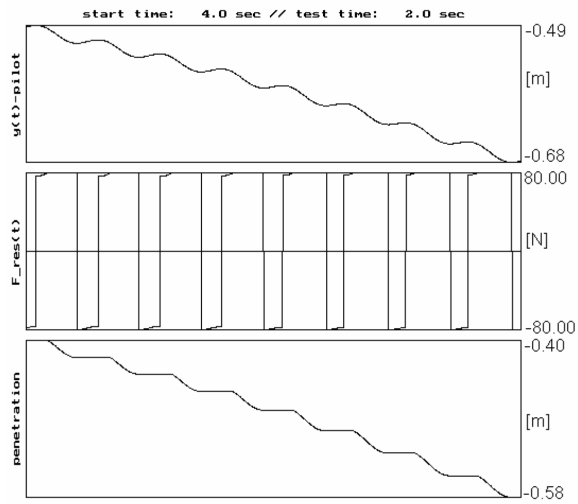


Figure 3. Diagrams for pile deplacement,penetration and total resistant force,for a steady state (at 4 sec. after the movement begin).

For time period when the system work in transient state, pile penetration is null, because forces generate by pile movement don't exceed the total soil resistance.

In Figure 3 are cutted out a time period whole of 2 sec, but after other 4 sec beyond movement started.

In these diagrams appear only steady state of system time evolution.
In Figure 3 it can be observed that both the pile deplacement and the penetration have a linear down direction movement. Also, the resistant force present a constantly evolution in analysing time window.

## 5. Conclusions

For this numerical application the limit forces of soil, $P_{21}$ and $P_{22}$, was considered like constant values, equals with medium resistance from experimental determinations.
To improve to performances of this model is necesary to consider the real evolution of soil resistances, through numerical function (real values experimental estimated) or analytical function obtained from experimental data, through numerical interpolation.
This work was proved that the Bingham reological model estimate very well the soil behaviour under the loads.
But, for better results it is necessary to estimate both the damping coefficient and the analitical expression for the dependence upon the damping force and the pile velocity which will modelling more realistical the vibrodriving phenomena, and the soil rigidity on the pile edgewise side and on the top of the pile.
By holding a lot of experimental data, and by numerical processing of them, it can obtained a very useful values for the parameters necesary to tuning on the model of the vibratory pile driving. Thus, the numerical analysis, based on the proposed model it can be use to predicte the behaviour of piles, along the driving process, actually before the real action.

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