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ABOUT DYNAMIC STABILITY ANALYSIS OF THE POWER REGULATORS WITH EXACTLY HYPERBOLIC CHARACTERISTIC

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ABSTRACT

Power regulators are attached devices of hydraulic pumps with variable capacity. Theses have the means to modify the pump capacity depending of the pressure from the circuit served by the pump. In this way, it result that the pump hydrostatic power will be mantained at the nominal working point value from external characteristics of driving unit motor. Thus, the product beetween pump momentary flow and the momentary pressure from pump circuit, give the same value all the time, the mathematical law for this phenomenon beeing a equilater hyperbola. In this paper the authors present the mathematical model for the control system ensemble, composed from pump balance system, dog watch system and power regulator unit.

1. Introduction

For modeling of the power self-acting regulation process it consider the drive system from Figure 1,

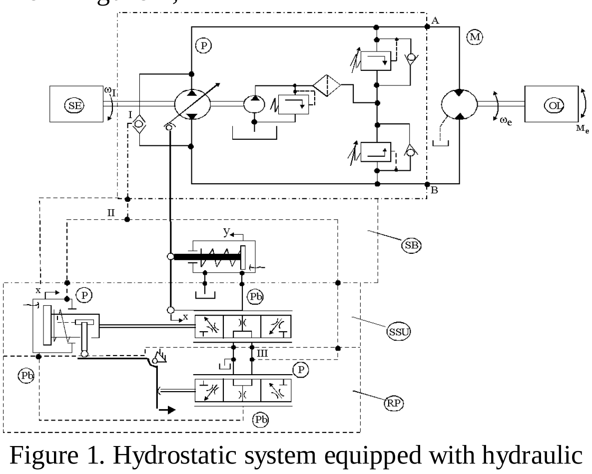

Figure 1. Hydrostatic system equipped with hydraulic servo-driving power regulator
OL - working tool; SB - the pump capacity variation system; SSU - the tracing servo-system; RP - power regulator; SE - primary energy source

where the P pump of the system is equipped with a hydraulic servo-driving power regulator (torque), and the M hydraulic motor have the fixed capacity and directly drive the working tool.

For writing the mathematical model of the system it was considered the components of the assembly, and it was elaborating the individual mathematical model for each part.

It consider that the system are composed from two mainly parts:

- execution branch - composed from the pump (P), the motor (M), the working tool (OL) and the hydraulic circuit.
- driving branch - composed from the regulator mechanism for pump capacity (SB), the tracing system (SSU), the power regulator (RP), the reaction links, and the driving hydraulic circuit.

2. The Mathematical Model of the Execution Branch

The hidraulic agent momentary flow given by the pump into the action system is

$$Q_p = \mathrm{H}_p \frac{V_{op}}{2\pi}\omega_I = Q_{op}\frac{S_m - x}{S_m} \qquad (1)$$

where: H_p - the pump capacity regulation factor; ω_I - the angular speed at the pump spindle; V_{op} - the maximum capacity of the pump; Q_{op} - the maximum flow of the pump; S_m - the maximum drive of the power regulator; x - the momentary drive of the power regulator.
The momentary flow of the self-acting system motor (M) will be

$$Q_M = \frac{V_{OM}}{2\pi}\omega_e \qquad (2)$$

where V_{OM} is the hydraulic motor capacity.
Consider the lost flow in the system through the constructive clearances of the pump and the hydraulic motor, of the shape $Q_c = \alpha_{PM}\, p$, and the flow disappear through the hydraulic agent compressibility, of the shape $\alpha_C \dot{p}$, where $\alpha_C = V_o / E_r$, result the flow equation

$$Q_{OP}\left(\frac{S_m - x}{S_m}\right) = \frac{V_{OM}}{2\pi}\omega_e + \alpha_{PM}\, p + \alpha_C \dot{p} \qquad (3)$$

The dynamic equilibrium equation of torque at the hydraulic motor spindle is

$$J_R \frac{d\omega_e}{dt} + \left(\gamma_M + \frac{V_{OM}}{2\pi}\delta_M\right)\omega_e + M_e = \frac{V_{OM}}{2\pi} p \qquad (4)$$

where J_R - the inertia moment reduced to the hydraulic motor spindle for all the components which have angular movement produced by the motor; the paranthesis means the torque losses which are proportional with the angular speed; M_e - the work tool resistant moment; p - the momentary pressure in the active branch of the execution hydraulic circuit.
Using the next notations

$$\alpha_{11}\alpha_{23} = A_1\, ;\ (\alpha_{23}\alpha_{12} + \alpha_{22}\alpha_{11}) = A_2$$

$$(\alpha_{21} + \alpha_{12}) = A_3\, ;\ \alpha_{22}\alpha_{13} = A_4\, ;\ \alpha_{23}\alpha_{13} = A_5$$

$$\frac{2\pi}{V_{OM}} J_R = \alpha_{11}\, ;\ \frac{2\pi}{V_{OM}}\left(\gamma_M + \frac{2\pi}{V_{OM}}\delta_M\right) = \alpha_{12}$$

$$\frac{2\pi}{V_{OM}} = \alpha_{13}\, ;\ \frac{V_{OM}}{2\pi Q_{OP}} = \alpha_{21}$$

$$\frac{\alpha_{PM}}{Q_{OP}} = \alpha_{22}\, ;\ \frac{\alpha_{OP}}{Q_{OP}} = \frac{V_o}{E_r Q_{OP}} = \alpha_{23}$$

and processing the equations (1) to (4) result the next two equations set that describe the working of the first mean component of the presented system

$$p = \alpha_{11}\dot{\omega}_e + \alpha_{12}\omega_e + \alpha_{13}M_e \qquad (5.1)$$

$$1 - \frac{x}{S_M} = A_1\ddot{\omega}_e + A_2\dot{\omega}_e + A_3\omega_e + A_4 M_e + A_5\dot{M}_e \qquad (5.2)$$

3. The Mathematical Model for the Command Branch

The system for pump weighing is presented in Figure 2. It was considered that the next specific quantities are determinables: A_b - the aria of the pump weighing piston; r - the lenght of the weighing lever; m_b - the reduce mass of the mechanism on the stem direction; k_b - the spring rigidity; y_{ob} - the intial spring compression; y - the momentary deplacement of the system, which determine the momentary capacity of the pump; $\alpha_{\max}$ - the maximum angle of pump weighing; $\alpha_{\min}$ - the limit (minimal) angle of pump weighing; k_{fb} - the viscous friction coefficient for weighing mechanism.

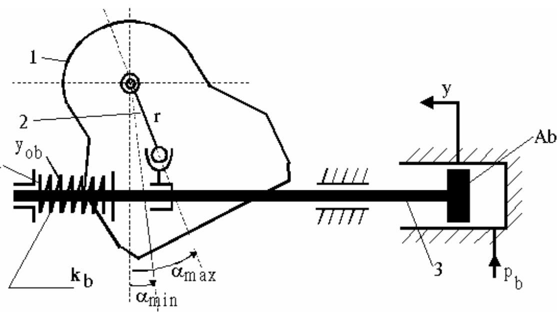

Figure 2. Pump capacity variation diagram
1- pump block; 2- balancing lever; 3- the balancing mechanism piston and the rod; 4- spring.

Taking into account the previous statements, the dynamic equilibrium equation, on the weighing mechanism stem direction will be

$$m_b\ddot{y} + k_{fb}\dot{y} + k_b(y_{ob} + y) = F_r + F_h \qquad (6)$$

where F_r - the returning force at the minimal angle for the pump block; F_h - the hydrostatic pump weighing force.
Considered the notations

$$2\xi_b\omega_b = k_{fb}/m_b\ ;\ \omega_b^2 = k_b/m_b\ ;$$

$$\alpha_{1b} = A_b/m_b\ ;\ \alpha_{2b} = A_p\lambda_1 z''/m_b\ ,$$

the equation (6) become

$$\ddot{y} + 2\xi_b\omega_b\dot{y} + \omega_b^2 y = \alpha_{1b} p_b + \alpha_{2b} p \qquad (7)$$

The servo-tracing system is composed from proportional distributor with moveable external hull, linked through the reaction levers with the weighing mechanism stem. The drawers of the proportional distributor is actuating by a differential piston driving by the pressure (p) from execution branch and the pressure (p_s) resulted from the power regulator proportional distributor.

It was considered that the next specific quantities are determinables: d_s - the diameter of the drawers; A_{1b} , A_{2b} - the aries of the differential piston; k_s , x_{os} - the rigidity and the initial compression of the spring; x - the momentary deplacement of the system; a_1, b_1, a_2, b_2 - the lengths of the reaction levers; y - the momentary deplacement of the weighing system; m_s - the reduce mass on the deplacement direction of moveables elements.

Taking into account the previous affirmations, the dynamic equilibrium equation, on the movement direction of the mechanism, is

$$m_s\ddot{x} + k_{fs}\dot{x} + k_s(x_{os} + x) = A_{2s} p_s - A_{1s} p \qquad (8)$$

It was considered that the p_o and p_{os} pressure values coresponding the static equilibrium of the system, and the p and p_s are pressures which variances around the static equilibrium values. Also it was maded the next notations

$$2\xi_s\omega_s = k_{fs}/m_s \; ; \; \omega_s^2 = k_s/m_s \; ;$$

$$\alpha_{1s} = A_{2s}/m_s \; ; \; \alpha_{2s} = A_{1s}/m_s$$

In this conditions, the equation (8) become

$$\ddot{x} + 2\xi_s\omega_s\dot{x} + \omega_s^2 x = \alpha_{1s} p_s - \alpha_{2s} p \qquad (9)$$

For the reaction levers system and for the distributor movement command lever it will be consider only the kinematical link realized between the weighing system and the distributor jack drive.

The power regulator is presented in Figure 3 and is composed from a proportional distributor and a levers system which realize the product of the proportional parameters with the pressure and the flow of the pump.

Also, for this component, it was considered that the next quantities are determinables: d_p - the diameter of the drawers; k_o - the spring rigidity; z_o - the initial compression of the spring; k_{fp} - the damping coefficient; z - the momentary deplacement of the proportional distributor; m_p - the reduce mass on the z deplacement direction; a_r - the aria of the head-on piston surface.

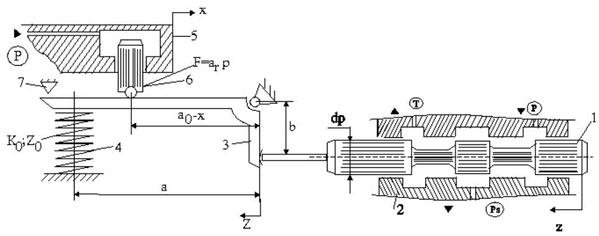

Figure 3. Power regulator diagram

1 - the distributor drawers; 2 - the distributor block; 3 - the power regulator rod; 4 - spring; 5 - the servo-tracing differential piston.

The dynamic equilibrium equation, reduce on the z movement direction of a proportional distributor drawers, is

$$m_p\ddot{z} + k_{fp}\dot{z} + k_o\left(z_o + \frac{a}{b}z\right)\frac{a}{b} = \frac{a_r}{b}p(a_o - x) \qquad (10)$$

Considered that the p_o value of the pressure corresponding the static equilibrium, and making the next notations

$$2\xi_p\omega_p = k_{fp}/m_p \; ; \; \omega_p^2 = \frac{k_o a^2}{b^2 m_p} \; ;$$

$$\alpha_{1p}^* = a_r a_o/b \; ; \; \alpha_{2p}^* = a_r/b \, ,$$

the equation (10) become

$$\ddot{z} + 2\xi_p\omega_p\dot{z} + \omega_p^2 z = \alpha_{1p}^* p - \alpha_{2p}^* p\, x \qquad (11)$$

In equation (11) the second member of the right side is a non-linear term because is a product of variables. In this sense the authors used the product development around the stationary values x_o and p_o

$$\alpha_{2p}^* p x = \alpha_{2p}^* p_o x + \alpha_{2p}^* x_o p \qquad (12)$$

Taking into account the previous equation, the equation (11) become

$$\ddot{z} + 2\xi_p\omega_p\dot{z} + \omega_p^2 z = \alpha_{1p} p - \alpha_{2p}\, x \qquad (13)$$

The driving hydraulic system assume the command signal from execution circuit with the help of circuit selector device, placed in node I. The flow equation for this node (I) is

$$Q_p = Q_E + Q_c \tag{14}$$

where the Q_E represent the pump flow assumed from the execution branch and is marked by the right term of equation (3); Q_c represent the flow assumed from the command branch of the system, and have the the expresion given by

$$Q_c = Q_{c1} + Q_{c2} \tag{15}$$

The Q_{c1} term is the agent flow absorbed from the direct branch of servo-tracing system, respectively on A_{1s} surface of a differential piston and from no. 6 piston movement.
The mathematical equation for Q_{c1} term is

$$Q_{c1} = A_{1s}\dot{x} + \frac{V_{os}}{E_r}\dot{p} + a_r\frac{b}{a}\dot{z} + k_{os}p \tag{16}$$

The Q_{c2} term represent the hydraulic agent flow assumed by the proportional distributors, which are evaluated in the *I* node.
This term is a sum of the flow assumed by the proportional distributor of a power regulator, and the flow assumed by the proportional distributor of a tracing system.
The both equations of the flows that composed the Q_{c2} expression are non-linear because contained the product of variables.
After liniarization, the Q_{c2} equation is

$$Q_{c2} = A_{2s}\dot{x} + \frac{V_{os}^*}{E_r}\dot{p}_s + k_{os}p_s + A_b\dot{y} + \frac{V_{ob}}{E_r}\dot{p}_b + k_{ob}p_b \tag{17}$$

Processing the equations (14) - (17), and making some notations which simplify the writing, result the next set of flow equations

$$\begin{aligned} Q_{op} = {} & \alpha_{21}\omega_e + \beta_{11}x + \beta_{12}p + \beta_{13}\dot{p} + A_{1s}\dot{x} - \\ & \beta_{14}p_s + \beta_{15}p_b + \beta_{16}z + \beta_2\dot{z} + \beta_{17}y - \beta_{18}p_T \end{aligned} \tag{18.1}$$

$$\beta_{21}z + \beta_{22}p = A_{2s}\dot{x} + \beta_{23}p_s + \beta_{1s}\dot{p}_s - \beta_{24}p_T \tag{18.2}$$

$$\begin{aligned} \beta_{31}x + \beta_{32}p = {} & \beta_{33}y + A_b\dot{y} + \\ & \beta_{34}p_b + \beta_{1b}\dot{p}_b - \beta_{35}p_T \end{aligned} \tag{18.3}$$

4. Summary

Consider the previous equations (5.1), (5.2), (7), (9), (13), (18.1) - (18.3), the mathematical model of the dynamic stability of power regulators is perfect formulated. For analitical analysis of this eight equations system, with the next eight unknown variables: *x(t)*, *y(t)*, *z(t)*, *p(t)*, $p_b(t)$, $p_s(t)$, $M_e(t)$ and $\omega_e(t)$, it can be used a liniarized formulations for the non-linear term of equations.
In case of numerical analysis the authors recommends using of the first formulation for all the equations (with non-linear terms).
This fact lead to the results that describes the considered variables evolutions with less errors, comparative to the analitical solutions and with the real behaviour of the system.
The mathematical model presented in this paper offer the opportunity the analyse the dynamic behaviour of the system, in the next situations:

- after equations liniarization, using the transfer functions of the system;
- with non-linear model, through the numerical analysis using the specifical software;
- with dedicated software, maded with the help of advanced programming languages.

Regardless the way to study the system behaviour, it could be obtain the influences of the constructive parameters, of the hydraulic medium parametrs, of the energetical parameters of the driving which will used this system.

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