THE DYNAMICS ANALYSIS OF THE FORCED STEADY-STATE FOR THE VERTICAL TOWERS CHILLERS WITH INTERNAL AXIAL FLOW FAN

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ABSTRACT

This study presents some of the phases of the physical and mathematical models' elaboration of the vertical tower chillers with a single axial flow fan, which are used in social and civil buildings to prepare conditioning air with medium and high flow. It has a real utility in designing, in manufacturing as well as in the exploitation of this type of equipment.

1. Introduction

The chillers are the main part of the equipment for air conditioned in most civil and public buildings, also for refrigerating storage yards or for buildings with controlled medium conditions (temperature, humidity, dust, a.s.o.).

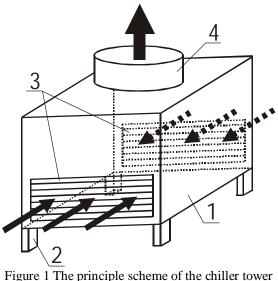


Figure 1 The principle scheme of the chiller tower with internal axial flow fan 1-chiller case 2-tower's supports 3-air inlet ports 4-outlet port

For all types of chillers, the part which moves the air or the agent of freezing (and/or heating in the case of reversing machines such as heat pumps) is the ventilator (no matter the type: axial fan, centrifugal/bladed fan). At the same time, the ventilator increases the pressure of the fluid and assures the necessary flow for the climatizer.

The figure no. l shows the scheme of a chiller tower with two sided inlet ports and one vertical outlet port. The ventilator isn't shown but, in most cases the vertical towers have one axial fan inside the case.

2.Physical model of the chiller tower

The *figure no.* 2 shows the calculus model for the chiller tower beared on four identical elastic

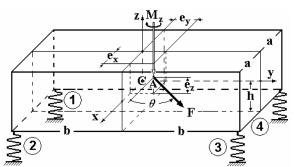


Figure 2 The calculus scheme of the tower with four below elastic suports(1, 2, 3, 4)C-the weight center of the assembly of the tower (including driving motor)A-the rotation center of the eccentric masses supports (it considers that the supports have no damping or these ones are almost negligible).

For constructive reasons as well as to simplify the calculus, the tower is modeled as a rigid body (parallelepipedon with $2a \times 2b$ base dimension) with six freedom degree and beared simmetrically. According these reasons, the most adequate cartesian axis system is the central and principal system, the inertia features beeing known as follows:

m -the total mass of the tower

$$J_x, J_y, J_z$$
-the moments of inertia for

the central and principal axis **Cx**, **Cy**, **Cz** Also it considers that the four supports are liniar elastic for the three axes (*figure no. 3*),

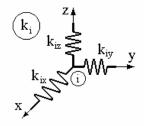


Figure 3 The principle scheme of the triorthogonal ellastic support
i-the ordinal number of the ellastic support
k_{ix}, k_{iy}, k_{iz}-the linear elastic characteristic of the support no. i for directions x, y and z

the characteristic beeing (k_{ix}, k_{iy}, k_{iz}) $i = \overline{1,4}$.

3. The model of the ventilator

The ideal ventilator for the chiller has a very good balanced rotor with the rotation axis mounted exactly on the vertical **Cz** axis of symmetry of the tower. Anyway, due to the technological conditions of the fabrication and dynamic unbalancing of the rotor on the one hand and the differences between the two axis result of the mounting on the other hand, on the assembly tower-ventilator is acting an inertial rotational force

$$F = m_0 r \omega^2, \tag{1}$$

where : $m_0 r$ is the statical moment of the unbalanced rotor;

 ω - rotational speed of the rotor;

$$\theta = \omega t$$
 - rotation angle of the rotor.

On the hypothesis of the parallelism between the axis of the tower and the axis of the ventilator, the force F is acting in point A (*figure no. 2*), which has the grid coordinates

$$A(e_x, e_y, e_z), \qquad (2)$$

and the position vector:

$$\bar{r}_A = e_x \bar{i} + e_y \bar{j} + e_z \bar{k} \tag{3}$$

The vectorial formula for the horizontal inertial force is :

$$\overline{F} = m_0 r \omega^2 \cos \theta \overline{i} + m_0 r \omega^2 \sin \theta \overline{j} \qquad (4)$$

The moment of the force F as against the point **C** is

$$\overline{M}_C(\overline{F}) = \overline{r}_A \times \overline{F} = \begin{vmatrix} \overline{i} & \overline{j} & k \\ e_x & e_y & e_z \\ m_0 r^2 \cos \theta & m_0 r^2 \sin \theta & 0 \end{vmatrix},$$

or :

$$\overline{M}_{C}(\overline{F}) = m_{0}r\omega^{2}(-e_{z}\sin\theta\overline{i} + e_{z}\cos\theta\overline{j}) + m_{0}r\omega^{2}(e_{x}\sin\theta - e_{y}\cos\theta)\overline{k}$$
(5)

The acting moment of the electrical motor which is driving the fan is modeled by M_7 .

4. The mathematical model of the chiller tower with one axial fan

For mathematical model it use the displacements as the generalised coordinate:

- X -side slip movement
- Y -advance movement
- Z -lift up movement
- φ_{χ} -pitching rotation

 φ_{v} -rolling rotation

 ϕ_7 -swing rotation

From the Lagrange equations of second species for the non disturb force vibrations of the system, the decoupling movement equations are the next:

a) for side slip movement and rolling rotation

$$\begin{cases} mX + 4k_x X - 4hk_x \varphi_y = 0\\ J_y \overline{\varphi}_y - 4hk_x X + 4 \left(h^2 k_x + a^2 k_z \right) \varphi_y = 0 \end{cases}$$
(6)

$$\begin{cases} m\Psi + 4k_yY + 4hk_y\varphi_x = 0\\ J_x\overline{\varphi}_x + 4hk_yY + 4(b^2k_z + h^2k_y)\varphi_x = 0 \end{cases}$$
(7)

c)for the lift up movement

$$n\mathbf{Z} + 4k_z Z = 0 \tag{8}$$

d)for the swing rotation

$$J_z \overline{\boldsymbol{\phi}}_z + 4 \left(a^2 k_y + b^2 k_x \right) \boldsymbol{\varphi}_z = 0 \tag{9}$$

The poulsations of the noncoupling movements for each "direction" are:

$$p_X = 2\sqrt{\frac{k_X}{m}} \tag{10}$$

$$p_Y = 2\sqrt{\frac{k_y}{m}} \tag{11}$$

$$p_Z = 2\sqrt{\frac{k_z}{m}} \tag{12}$$

$$p_{\mathbf{\varphi}_{x}} = 2\sqrt{\frac{b^{2}k_{z} + h^{2}k_{y}}{J_{x}}} \tag{13}$$

$$p_{\mathbf{\phi}_y} = 2\sqrt{\frac{h^2 k_x + a^2 k_z}{J_y}} \tag{14}$$

$$p_{\varphi_z} = 2\sqrt{\frac{a^2k_y + b^2k_x}{J_z}} \tag{15}$$

The natural frequencies of the decoupling movements of the chiller tower are

a) for side slip movement and rolling rotation

$$f_{1} = \frac{\sqrt{\frac{l}{2}} \left[p_{X}^{2} + p_{\varphi_{y}}^{2} - \sqrt{\left(p_{X}^{2} - p_{\varphi_{y}}^{2} \right)^{2} + 4\alpha_{I}\alpha_{2}} \right]}{2\pi} (16)$$

$$f_{2} = \frac{\sqrt{\frac{l}{2}} \left[p_{X}^{2} + p_{\varphi_{y}}^{2} + \sqrt{\left(p_{X}^{2} - p_{\varphi_{y}}^{2} \right)^{2} + 4\alpha_{I}\alpha_{2}} \right]}{2\pi} (17)$$

b) for advance movement and pitching rotation

$$f_{3} = \frac{\sqrt{\frac{1}{2} \left[p_{Y}^{2} + p_{\varphi_{X}}^{2} - \sqrt{\left(p_{Y}^{2} - p_{\varphi_{X}}^{2} \right)^{2} + 4\beta_{I}\beta_{2}} \right]}{2\pi} (18)$$

$$f_{4} = \frac{\sqrt{\frac{1}{2} \left[p_{Y}^{2} + p_{\varphi_{X}}^{2} + \sqrt{\left(p_{Y}^{2} - p_{\varphi_{X}}^{2} \right)^{2} + 4\beta_{I}\beta_{2} \right]}}{2\pi}$$
(19)

c) for the lift up movement

$$f_5 = \frac{p_Z}{2\pi} = \frac{1}{\pi} \sqrt{\frac{k_z}{m}}$$
(20)

d)for the swing rotation

$$f_6 = \frac{p_{\varphi_z}}{2\pi} = \frac{1}{\pi} \sqrt{\frac{a^2 k_y + b^2 k_x}{J_z}} , \qquad (21)$$

where the relations between the poulsations and the frequncies are

$$p_i = 2\pi f$$
 $i = 1,6$

and the used notations means:

$$\alpha_I = -\frac{4}{m}hk_x \quad \alpha_2 = -\frac{4}{J_y}hk_x \quad (22a)$$
$$\beta_I = -\frac{4}{m}hk_y \quad \beta_2 = -\frac{4}{J_x}hk_y \quad (22b)$$

5.The mathematical model of the forced steady-state vibrations

If it considers that the actions on the tower are the inertial force \overline{F} and the reaction moment \overline{M}_z , the generalized forces are:

$$Q_X^F = m_0 r \omega^2 \cos \omega t \tag{23a}$$

$$Q_Y^F = m_0 r \omega^2 \sin \omega t \tag{23b}$$

$$Q_Z^F = 0 \tag{23c}$$

$$Q_{\mathbf{\varphi}_{x}}^{F} = -e_{z}m_{0}r\omega^{2}\sin\omega t \qquad (23d)$$

$$Q_{\phi_y}^F = e_z m_0 r \omega^2 \cos \omega t \tag{23e}$$

$$Q_{\varphi_z}^F = -M_z(t) + m_0 r \omega^2 (e_x \sin \omega t - e_y \cos \omega t) (23f)$$

The differential equations for the forced vibrations are obtained from Lagrange equations of second species: a)side slip movement and rolling rotation (24)

$$\begin{cases} m\overline{X} + 4k_xX - 4hk_x\varphi_y = m_0r\omega^2\cos\omega t\\ J_y\overline{\varphi}_y - 4hk_xX + 4(h^2k_x + a^2k_z)\varphi_y = e_zm_0r\omega^2\cos\omega t\\ \mathbf{b}) \text{advance movement and pitching rotation (25)} \end{cases}$$

$$\begin{cases} m\overline{P} + 4k_yY + 4hk_y\varphi_x = m_0r\omega^2 \sin \omega t \\ J_x\overline{\varphi}_x + 4hk_yY + 4(b^2k_z + h^2k_y)\varphi_x = -e_zm_0r\omega^2 \sin \omega t \\ \mathbf{c}) \text{ lift up movement} \end{cases}$$

 $m\vec{Z} + 4k_z Z = 0$

$$J_{z}\overline{\phi}_{z} + 4\left(a^{2}k_{y} + b^{2}k_{x}\right)\phi_{z} =$$

$$= -M_{z}(t) + m_{0}r\omega^{2}\left(e_{x}\sin\omega t - e_{y}\cos\omega t\right)$$
(27)

From above equations it's obvious that the disturbed "directions" are X, Y, φ_x , φ_y and φ_z ; the amplitudes of harmonical forced steady-state vibrations for each "direction" are:

$$A_{X} = \frac{m_{0}r\omega^{2} \left[\left(p_{\varphi_{y}}^{2} - \omega^{2} \right) - \alpha_{2}e_{z} \right]}{m \left(p_{1}^{2} - \omega^{2} \right) \left(p_{2}^{2} - \omega^{2} \right)}$$
(28)

$$A_{\varphi_{y}} = \frac{m_{0}r\omega^{2} \left[\alpha_{I} + \left(p_{X}^{2} - \omega^{2} \right) e_{z} \right]}{J_{y} \left(p_{I}^{2} - \omega^{2} \right) \left(p_{2}^{2} - \omega^{2} \right)}$$
(29)

$$A_{Y} = \frac{m_{0}r\omega^{2} \left[\left(p_{\varphi_{X}}^{2} - \omega^{2} \right) - \beta_{2}e_{z} \right]}{m \left(p_{3}^{2} - \omega^{2} \right) \left(p_{4}^{2} - \omega^{2} \right)}$$
(30)

39

(26)

$$A_{\varphi_{X}} = -\frac{m_{0}r\omega^{2} \left[e_{z} \left(p_{Y}^{2} - \omega^{2} \right) - \beta_{I} \right]}{J_{x} \left(p_{3}^{2} - \omega^{2} \right) \left(p_{4}^{2} - \omega^{2} \right)}$$
(31)

The movement around the vertical axis (swing rotation) is given by the solution of the equation (27) and depends of the function $M_z(t)$. If the rotational speed of the ventilator is constant, the moment of the ventilator drive is also constant and the solution is

$$\varphi_{z}(t) = -\varphi_{0} + \varphi_{zf}(t) , \qquad (32)$$

where : ϕ_0 is the angular displacement of the tower from static balance position ;

 $\varphi_{zf}(t)$ - the harmonical forced vibration which has the formula

$$\varphi_{zf}(t) = A_{\varphi_z} \sin(\omega t + \theta_0)$$
(33)

The amplitude of forced vibrations is

$$A_{\varphi_z} = \frac{\varepsilon m_0 r \omega^2}{J_z \left(p_6^2 - \omega^2 \right)}, \qquad (34)$$

where $\varepsilon = \sqrt{e_x^2 + e_y^2}$ is the distance between the axis of the tower and the axis of the ventilator.

6. Conclusions

The constructive model of chiller tower that this study proposes can led to achieving a rigorous physical model and as calculus, to evidence the dynamic aspects related to the functioning of this type of equipment.

It can be demonstrated that, if some dimensional and functional conditions are fulfilled, under certain working hypotheses, the natural vibrations of different "directions" are decoupled and some of the undesired forced vibrations may be annulled. In order to reduce the amplitudes of steadystate forced vibrations, it's necessary to take into account the next technological (manufacture, test, ajustments, mounting) measures:

-a good static and dynamic balancing of the rotors of the motors and ventilators (technical and economical optimum if possible);

-the best alignment of the axis of the ventilator by vertical axis of the chiller tower.

Under the hypothesis of decoupling of all motions in four subsystems and considering that the system tower-ventilator has small oscillations, it can be demonstrated that these vibrations are quasiharmonical, with the pulsation equal to that of the driving motor.

7. References

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