THE ANNALS OF “DUNAREA DE JOS” UNIVERSITY OF GALATI
FASCICLE XIV MECHANICAL ENGINEERING, ISSN 1224 – 5615
2004

MATHEMATICAL MODEL FOR DYNAMIC BEHAVIOUR ANALYSIS OF THE PASSIVE ISOLATION SYSTEMS

Asist. univ. ing. drd. Silviu NASTAC
Universitatea “Dunarea de Jos” Galati

ABSTRACT

In this paper the authors present a set of mathematical models, usable to simulate and analyse the static and dynamic behaviour aspects, both for the passive anti-vibrational and anti-seismical systems, and for all the machines and equipments that are moving on the unarranged roads, in the length time of a technological process. For all the considered modells the authors have write the characteristic system of movement equations and present the numerical simulation results, with taking into account the real equipments input data.

1. Introduction

The problematics of the anti-vibrational or anti-seismical isolation for the sensitives machineries and equipments suppose the presence of the specialised systems for reduce or eliminate the transmission phenomenon of the vibrations produced by the certain source, to the machinery or equipment with the necessary continous operating state, regardless to the environment conditions. Taking into account the operating way of the isolation ssytems, these could be divided into the next classes:

- passive systems - the functional parameters that global characterised the isolation capacity, are imposed only by the stiffness and dissipative characteristics of the elastic components of the isolation systems
- active (adaptive) systems - these systems contained, beside on the effective isolation components, the complex sub-system for aquisition, processing and adjustment the elastic and dissipative characteristics of the isolators; entire the components aquire continously the specific parameters of the vital equipments and make the necessary adjustments, on the real time, with the aim of the reducing the vibration transferability factor

In this paper are treats only the passive kinds of the isolation systems for vibrations and seismical waves. The working principle of this systems are based on the elastic capability of the rubber elastic elements to assume the exciting loads energy and transforming it into the potential energy.

2. Mathematical Model

The presented model have two degree of freedom: translation between Oy axis and angular movement along the center of gravity into the Oxy plane. This physical model is presented in the Fig. 1.

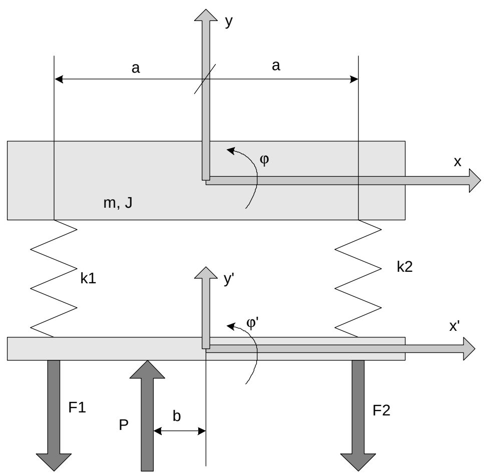


Figure 1. Physical model of isolation system

For write the characteristic equations of this model that considered the next arrays: matrix of the inertial characteristics M, generalized movements vector q, the stiffness matrix K and external loads vector L, with the next forms:

$$M = \begin{bmatrix} m & 0 \\ 0 & J \end{bmatrix} \quad (1)$$

$$q = \begin{bmatrix} y \\ \varphi \end{bmatrix} \quad (2)$$

$$K = \begin{bmatrix} (k1+k2) & (k1-k2)a \\ (k1-k2)a & (k1+k2)a^2 \end{bmatrix} \quad (3)$$

$$L = \begin{bmatrix} P \\ Pb \end{bmatrix} = P \begin{bmatrix} 1 \\ b \end{bmatrix} \quad (4)$$

With this notations the mathematical model of the system presented in figure 1, are

$$M\ddot{q} + K\,q = L \quad (5)$$

From eq. (3) it can be observed that both the system equations are coupled, and for separating them it must accomplish the next condition:

$$k1 = k2 = k \quad (6)$$

The authors considered that even all the elastic elements have the same type and operting caracteristic, in the real mode it could be differents, and of this motive, the link between the two variable k1 and k2 are:

$$k2 = k1\,\alpha = k\,\alpha \quad (7)$$

where α is the geometric non-linearity coefficient.

3. Numerical Simulation

For simulate the dynamic behaviour of the isolating system it was considered two kinds for the exciting force. First type was a harmonical time function, with constant magnitude (100 N) and frequency (5 Hz):

$$P(t) = A\sin(2\pi f\ t) \quad (8)$$

The second exciting force type was a time function which simulate a seismical wave, with a time length about 3,5 sec:

$$P(t) = 600e^{-1,3(t-1,7)^2} \left(0,3\sin(35t-59,5) + 0,25\sin(130t)\right) \quad (9)$$

The diagram for the last type of the exciting force are presented in the Figure 2.
For numerical simulation of the dynamic behaviour, the authors using the next values for the constants that appear in the mathematical model (Table 1).

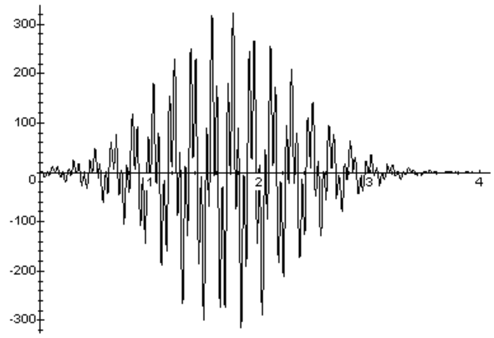
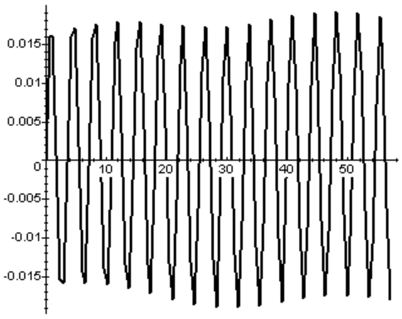


Figure 2. Temporal evolution of a virtual seismical wave

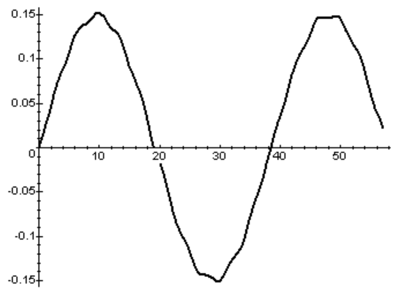
Table 1 Numerical values of the model

constant name	value	units
mass - m	100	kg
moment of inertia - J	10000	kgm^2
distance - a	1	m
distance - b	1	m
coefficient α	2	-
stiffness - k1	100	N/m

In the Figure 3 it is presented the time form of the vertical deplacement - a - and of the angular movement of the gravity center - b, with considering the case of the harmonical exciting force (eq. 8).
For the case of seismical wave form (eq. 9), the time evolution diagrams, for the same two deplacements [y(t), φ(t)], are presented in the Figure 4.
The authors was also analysis the influence of the coefficient α about the deplacements y(y) and φ(t), and was observed that at the same time of the α growing, bring up the modulation magnitudes both for the linear, and for the angular movements. Also, the eigen frequency for both deplacement types of the mass m acquire the high ranking values. These facts could be observed in the Figure 5. In this first case was used the harmonical type of exciting force (eq. 8).

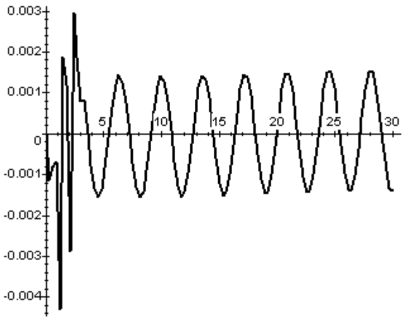


(a)

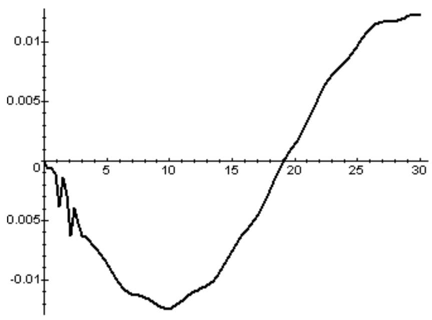


(b)

Figure 3. Temporal evolution of the system deplacements in case of harmonical exciting force
a- linear depl. in meters;
b- angular depl. in degrees; time in sec.



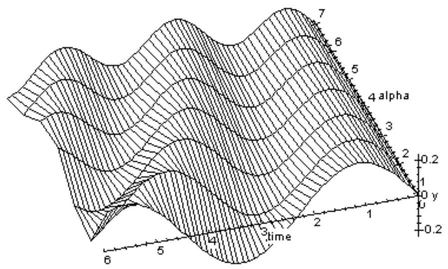
(a)



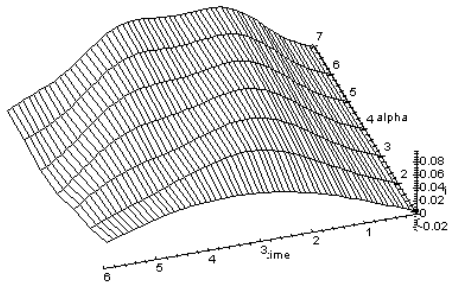
(b)

Figure 4. Temporal evolution of the system deplacements in case of seismic type exciting force
a- linear depl. in meters;
b- angular depl. in degrees; time in sec.

For the case of seismical wave form, the influences of the α coefficient about the deplacement shapes, could be view in the Figure 6. Analysing this last set of diagrams, it could be say that the influence of the geometrical non-linearity could be ignored in the time of external load acting, after that the system evolve to the stability in different ways, as the α coefficient has null or not.



(a)



(b)

Figure 5. The influences of the α coefficient about the system deplacements - *a- y(t); b- $\varphi(t)$*

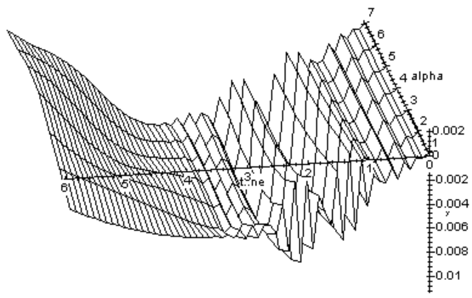
Eigen values and eigen vectors for the presented system was computed with the well known expresion:

$$\left(K-\lambda^2 M\right)\{\mu\}=\{0\} \tag{10}$$

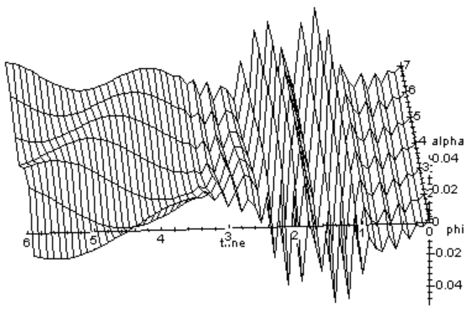
and was obtained the next values:

$$\lambda=\left[3{,}003363 \quad 0{,}026637\right] \tag{11}$$

$$\mu=\begin{bmatrix} 9{,}99435 & 0{,}33613 \\ -0{,}33613 & 0{,}999435 \end{bmatrix} \tag{12}$$



(a)



(b)

Figure 6. The influences of the α coefficient about the system deplacements - seismical load case
a- y(t); b- φ(t)

4. Concluding Remarks

Taking into account the numerical results, a part presented in this paper, and consider that the model will be tunned with experimental data, we could say that this physical and mathematical model are very useful for analysing the dynamic behaviour of the passive elastic isolation system against the nocive effects of the vibrations or seismical waves. Also, this model must be completed with a spectral analysis for obtained exctly informations about the spectral composition of the y(t) and φ(t) signals. This informations help designers to avoid the dangerous phenomenon of resonance, that could be appear not only for the eigen value of system frequency, and for the high values of external loads frequencies that concur with the superior harmonics of the system.

Acknowledgements

Numerical computations have been performed at the Numerical Simulation of Processes Departament, and the behavioural analysis of the proposed isolation systems have been performed at the Dynamic Analysis of the Machines Departament and Technological Equipments Departament, both from the Engineering Faculty of Braila - Research Center for Machines and Technological Equipments Mechanics.

References

[1] Bratu P. - *Izolarea si amortizarea vibratiilor la utilaje de constructii*, Ed. INCERC, Bucuresti, 1982.
[2] Bratu P., Bordea C. - *Sistem elastic poligonal pentru izolarea socurilor si vibratiilor transmise de echipamentele dinamice inglobate in constructii*, Buletinul stiintific al celei de a XXVI-a Conferinte Nationale de Mecanica Solidelor, Braila, 14-15 iunie 2002.
[3] Oproescu Gh., Nastac S. - *Elemente de modelare numerica*, Ed. Libertatea, Braila, 2000.
[4] Nastac S. - *Analiza numerica cu aplicatii in ingineria mecanica*, Ed. IMPULS, Bucuresti, 2004.
[5] Nastac S. - *Analiza miscarii sistemului elastic poligonal de protectie antiseismica*, Buletinul Stiintific al celui de al IX-lea Simpozion National de Utilaje pentru Constructii, Bucuresti, 11-12 decembrie 2003.
[6] Bratu P., Nastac S., Bordea C. - *Analiza dinamica a sistemelor elastice antiseismice cu acordare joasa pentru izolarea bazei constructiilor*, Buletinul stiintific al celei de a XXVII-a Conferinte Nationale de Mecanica Solidelor, Pitesti, 23-24 mai 2003.
[7] Bratu P., Nastac S., Bordea C. - *Analiza dinamica a sistemelor elastice de protectie antiseismica pentru echipamentele vitale din dotarea constructiilor de interes public*, Buletinul stiintific al celei de a XXVII-a Conferinte Nationale de Mecanica Solidelor, Pitesti, 23-24 mai 2003.
[8] Bratu P., Nastac S., Bordea C. - *Sistem elastic antivibratil si antiseismic in configuratie poligonala*, Buletinul Stiintific al celui de al VIII-lea Simpozion National de Utilaje pentru Constructii, Bucuresti, 12-13 decembrie 2002.
[9] Bratu P., Nastac S., Bordea C. - *Anti-seismic Protection Solution for vital equipments from public buildings endowment*, The International Conference Constructions 2003, Cluj-Napoca,16 -17 mai 2003.
[10] Nastac S., Bordea C. - *Cercetari experimentale asupra sistemelor pasive de protectie antiseismica in configuratie poligonala*, A III-a Sesiune de comunic`ri stiintifice a catedrei de mecanic` tehnica si mecanisme - SIMEC 2004, UTCB - Facultatea de Utilaj Tehnologic, Bucuresti, 26 martie 2004.
[11] Nastac S., Bordea C. - *Performantele func\ionale ale dispozitivelor si sistemelor pasive de protectie antiseismic`*, A III-a Sesiune de comunicari stiintifice a catedrei de mecanica tehnica si mecanisme - SIMEC 2004, UTCB - Facultatea de Utilaj Tehnologic, Bucuresti, 26 martie 2004.
[12] Bordea C., Nastac S. - *Sistem elastic in configuratie poligonala folosit la protectia antiseismica a echipamentelor*, Monitorul AROTEM, anul 7 nr. 4 octombrie - decembrie 2003.
[13] Nastac S. - *Non-Linear Stiffness Influences About the Dynamics of the Anti-Vibrational Passive Isolation Systems*, Romanian Journal of Acoustics and Vibration, anul 1 nr. 1 octombrie - 2004.