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ABOUT OF THE ANALYTICAL EXACTLY SOLUTIONS OF THE DIFFERENTIAL EQUATIONS OF THE NON-LINEAR OSCILLATORS.

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ABSTRACT

This work presents the manner of the solving of the non-linear differential equations from order II with the form $\ddot{y} + p(t)y = q(t)y^{1-2\alpha}$. *These equations can characterize the excited vibrations of the mechanical non-linear system for determined expressions of the function* $q(t)$ *as follows for the nonce.*

The vibration of the mechanical non-linear oscillator are characterized from the differential equation

$$m\ddot{y} + F_f(y,\dot{y}) + F_e(y,\dot{y}) = F(t)$$

where m =mass; F_f =friction force; F_e =elastic force; F =excitation force; y =displacement.

For this equation can be found an analytical exactly solution only in specially cases. A similar case will be resolved as follows.

The non-linear differential equation [1]

$$\ddot{y} + p(t) \cdot y = c \cdot y^{-2} \qquad (1)$$

has exact solutions of the form

$$y = \sqrt[\alpha]{u^2 + c \cdot \left(\frac{v}{w}\right)^2} \qquad (2)$$

when

$$y(t_0) = y_0 \neq 0,\ \dot{y}(t_0) = \dot{y}_0$$

for c an arbitrary constant and p(t) given. The functions u and v are independent solutions of the linear equations

$$\ddot{y} + p(t) \cdot y = 0 \qquad (3)$$

for which

$$u(t_0) = y_0,\ \dot{u}(t_0) = \dot{y}_0,\ v(t_0) = 0,\ \dot{v}(t_0) \neq 0$$

where their Wronskian

$$W = u \cdot \dot{v} - \dot{u} \cdot v = const. \neq 0\,.$$

With α integer finite and $\alpha \neq 0, \alpha \neq 1$ it is possible to show that

$$y = \left(u^{\alpha} + \frac{c}{\alpha - 1} \cdot \frac{v^{\alpha}}{w^2}\right)^{\frac{1}{\alpha}} \qquad (4)$$

is an exact solution of

$$\ddot{y} + p(t)\cdot y = q(t)\cdot y^{1-2\alpha} \qquad (5)$$

provided that u and v remain independent solutions of (3) and subject to the same conditions above, except that v_0 need not to be zero , and provided that

$$q(t) = c\cdot(uv)^{\alpha-2}. \qquad (6)$$

The proof is simple. Although q(t) clearly restricts the general class of nonlinear equations implied by (5), important physical problems occur with initial conditions such as to make the solutions (4) physically interesting. Moreover , it is interesting to regard the use of uv in q(t) as a method for generating nonlinear differential equations , for which an exact solution is (4) . The arbitrary choice of p(t) allows a wide range of possibilities. Taking

$$p(t) = \pm\omega^2 = const.$$

provides two immediate examples:

$$\ddot{y} + \omega^2 y = c_1(\sin\omega t\cdot\cos\omega t)^{\alpha-2}\cdot y^{1-2\alpha} \qquad (7)$$

$$\ddot{y} - \omega^2 y = c_1\cdot y^{1-2\alpha} \qquad (8)$$

having solutions

$$y = \left(a^{\alpha}\cdot\cos^{\alpha}\omega t + c_{\alpha}\cdot b^{\alpha}\cdot\sin^{\alpha}\omega t\right)^{\frac{1}{\alpha}}, \qquad (9)$$

$$y = \left(a^{\alpha}e^{\alpha\omega t} + c_{\alpha}^{1}b^{\alpha}e^{-\alpha\omega t}\right)^{\frac{1}{\alpha}} \qquad (10)$$

respectively,

where

$$c_{\alpha} = \frac{c_1}{(\alpha-1)\omega^2\cdot(ab)^{\alpha}},$$

$$c'_{\alpha} = \frac{c_1}{4(\alpha-1)\omega^2\cdot(ab)^{\alpha}}.$$

Constants a, b are determined by the initial conditions while c_1, equal to $c(ab)^{\alpha-2}$ is arbitrary.

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