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THE COEFFICIENT OF RESTITUTION IN THE PLASTIC DEFORMATION PROCESS

Conf.dr.ing. Ichim DRAGULIN
Asist.ing. Adrian LEOPA
Universitatea "Dunarea de Jos" din Galati

ABSTRACT

In the analysis of dynamic processes and plastic deformation which appear on the forging and die forging equipments dint the shocks (forging hammers and screw presses), it is use frequently theoretical study of de collision from theoretical mechanics.
Both the calculus of the efficiency of the forging hammer and the calculus of the damping system appear a coefficient "k" known as coefficient of restitution. This paper describes the physical pregnancy and the manner to evaluation of this coefficient of restitution.

1. Introduction

In theoretical study of head-on collision, we applied law of conservation of momentum and obtain the speeds of the body's after collision.
The three steps of collisions.

a) before collision
b) the phase of collision: compression and relaxing
c) after collision

2.Calculation elements

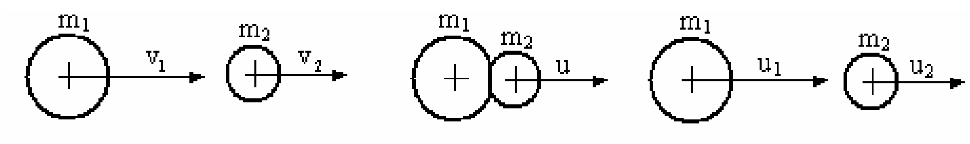
Coefficient of restitution is defined like ratio between relaxing impulse and compression impulse:

$$u_2 = v_2 + \frac{(v_1 - v_2)(1+k)}{1 + \frac{m_2}{m_1}}$$

and losing energy dint plastic deformation

$$\Delta E = \frac{m_1 m_2}{2(m_1 + m_2)}(v_1 - v_2)^2\left(1 - k^2\right)$$

In case of the forging hammer:
m_1 – represent mass of the ram
m_2 – represent mass of the anvil block
v_1 – represent the speed of the ram in impact moment
$v_2=0$



a b c

Figure 1 The steps of head-on collision

$$k = \frac{u_2 - u_1}{v_1 - v_2} = \frac{\left|u_{r_{12}}\right|}{\left|r_{r_{12}}\right|}$$

After collision:

$$u_1 = v_1 - \frac{(v_1 - v_2)(1+k)}{1 + \frac{m_1}{m_2}}$$

$$u_1 = v_1 - \frac{v_1(1+k)}{1 + \frac{m_1}{m_2}}; \quad u_2 = \frac{v_1(1+k)}{1 + \frac{m_2}{m_1}}$$

$$\Delta E = \frac{m_2}{(m_1 + m_2)}\frac{m_1 v_1^2}{2}\left(1 - k^2\right)$$ - effective work

if to note $\mu = \frac{m_2}{m_1}$

$$u_1 = v_1\left(1-\frac{1+k}{1+\frac{1}{\mu}}\right);\ u_2 = v_1\left(\frac{1+k}{1+\mu}\right)$$

$$\Delta E = E_1\frac{1-k^2}{1+\frac{1}{\mu}}$$

In functioning of the forging hammer are three types of the bounce:

- control bounce
- bounce on the piece
- bounce between anvils

The two latest bounce are extreme cases: bounce on the piece have maximum efficiency and bounce between anvils are errors to handle.
For the analysis we consider the second bounce namely upset forging of the cylindrical piece with the elasto-plastic behavior. The mass of the piece are neglected and she is included in the mass of anvil block.
The kinetic energy of the ram in the moment of impact:

$$E_1 = \frac{m_1 v_1^2}{2}$$

After strike the cylindrical piece is plastic deformed (fig. 2) with a unit strain

$$\varepsilon = \frac{H_0 - H}{H}$$

The deformations effect on the plastic properties of the technological material is cold-hardening (fig. 3). The coefficient of cold-hardening is

$$E_p \approx \frac{E}{100}$$

where E - coefficient of elasticity.
After deforming process the real unit strain

$$\in = ln\frac{H_0}{H} = ln(1+\varepsilon)$$

and the final forming stress is

$$\sigma_{ef} = \sigma_{C_0} + \in \cdot E_p$$

After compression phase, the elastic reaction of the deformed material is product between the volume of the deformed body and the elastic energy that are stored in compression times.

$$E_{el} = W_{el}\cdot V_{el} = A\cdot H\cdot\frac{\sigma_B^2}{2E}$$

In the time of compression the body's move together (both are the same speed).

$$u = \frac{m_1 v_1}{m_1 + m_2} = \frac{v_1}{1+\mu}$$

In the relaxing phase the equation of movement are:

$$\begin{cases} m_1 u_1 + m_2 u_2 = (m_1 + m_2)u \\ m_1 u_1^2 + m_2 u_2^2 = (m_1 + m_2)u^2 + 2E_{el} \end{cases}$$

using the notations:

$$\mu = \frac{m_2}{m_1};$$

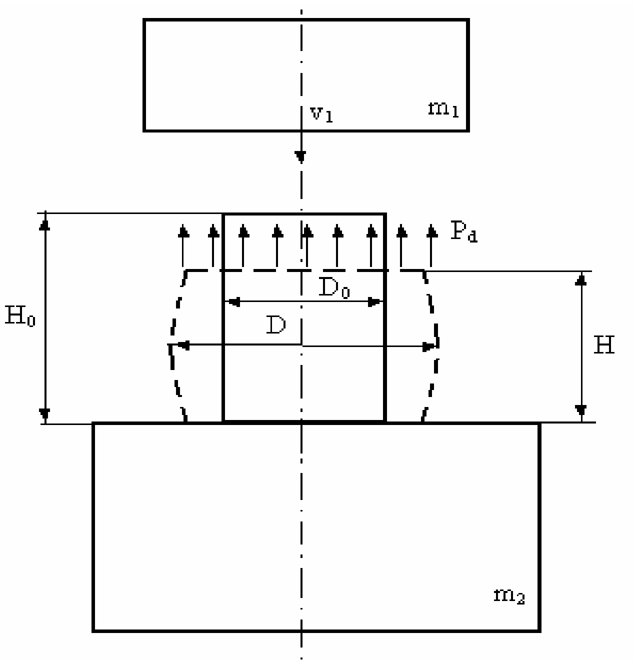


Figure 2. The upset forging process

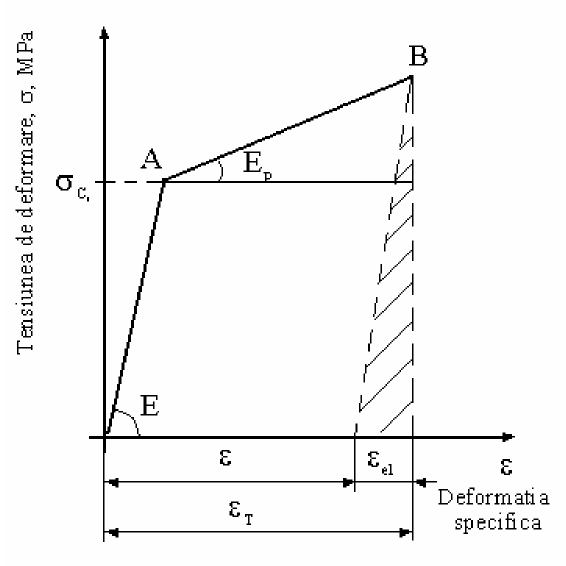


Figure 3 Diagram σ - ε

$$u = \frac{v_1}{1+\mu}; E_1 = \frac{m_1 v_1^2}{2}; m_1 = \frac{2E_1}{v_1^2}; \varepsilon = \frac{E_{el}}{E_1};$$

$$\begin{cases} u_1 + \mu u_2 = v_1 \\ u_1^2 + \mu u_2^2 = v_1^2\left(\frac{1}{1+\mu} + \frac{E_{el}}{E_1}\right) \end{cases}$$

$$u_1 + \mu u_2 = (1+\mu)\frac{v_1}{1+\mu}$$

and

$$u_1^2 + \mu u_2^2 = (1+\mu)\frac{v_1^2}{(1+\mu)^2} + \frac{E_{el}}{E_1} v_1^2$$

In final

$$\begin{cases} u_1 = \frac{v_1}{1+\mu}\left[1 - \sqrt{\varepsilon\mu(1+\mu)}\right] \\ u_2 = \frac{v_1}{1+\mu}\left[1 + \sqrt{\varepsilon\frac{1+\mu}{\mu}}\right] \end{cases}$$

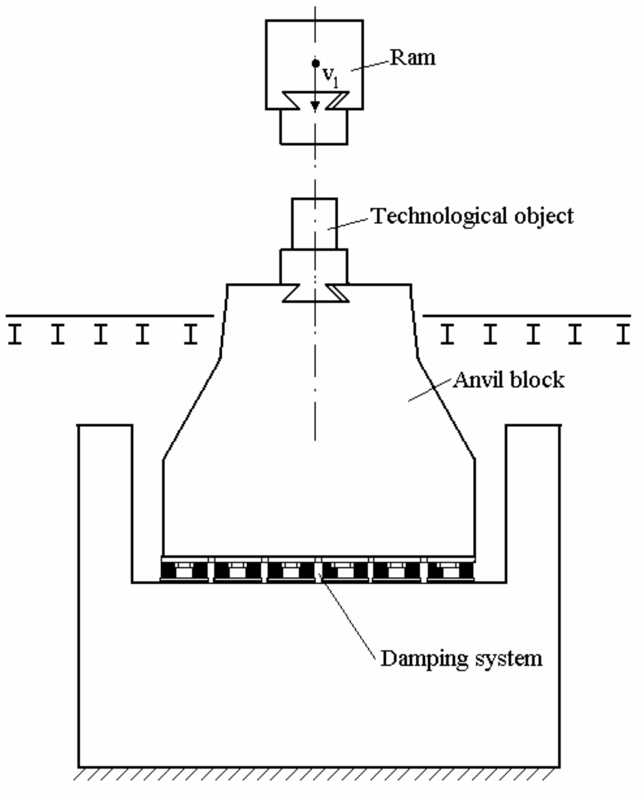


Figure 4. The basic diagram of forging hammer

According with definition of coefficient of restitution:

$$k = \frac{|u_2 - u_1|}{v_1} = \frac{\frac{v_1}{1+\mu}\left[\sqrt{\varepsilon\frac{1+\mu}{\mu}} + \sqrt{\varepsilon\mu(1+\mu)}\right]}{v_1}$$

$$k = \sqrt{\frac{E_{el}}{E_1}\left(1 + \frac{m_1}{m_2}\right)} \text{ or } k = \sqrt{\frac{E_{el}}{E_1}\left(1 + \frac{1}{\mu}\right)}$$

If the ratio $\frac{m_1}{m_2}$ is very small we can neglected this ratio and the coefficient of restitution become: $k \approx \sqrt{\frac{E_{el}}{E_1}}$; if $m_1 \approx m_2$ $k \approx 1.42 \cdot \sqrt{\frac{E_{el}}{E_1}}$.

The first case is encounter on free forging process and second case encounter on die forging process on forging hammer with duble action.

3. Conclusions

The coefficient of restitution is dependent, in the case of forging process, on elasto-plastic characteristics of materials (because the ratio $\frac{E_{el}}{E_1}$) and mass ratio between ram and anvil block.

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