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DYNAMIC ANALYZE FOR THE CHASSIS-CAB SYSTEM HAVING ELASTIC INTERMEDIATE LINK (ELASTIC INTERFACE)

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ABSTRACT

Among the machinery generating occupational diseases due to the vibrations transmitted to human body the portable hand-held construction equipment such as drill hammers and demolition hammers are classified. In order to characterize the level of vibrations transmitted to the human body and the exposure time, in the frame of ICECON Bucharest specific tests upon the hammer manufactured by the British Company Kango have been performed. The measurements have been carried out with the machine operating at rated working parameters, determining the accelerations in the three orthogonal directions defined by x, y and z axes. Aiming measuring the vibrations the Bruel & Kjaer chain has been used and figures illustrate the processed results as well as the permissible vibration limits for different exposure times.

1. Dynamic Models For The Construction Machinery Provided With Cab

1.1 Dynamic model in case of a construction machine that does not use vibrations as determining factor during the working process

1.1 Dynamic model in case of a construction machine that does not use vibrations as determining factor during the working process (excavators, loaders, graders, scrapers, technological transport equipment)

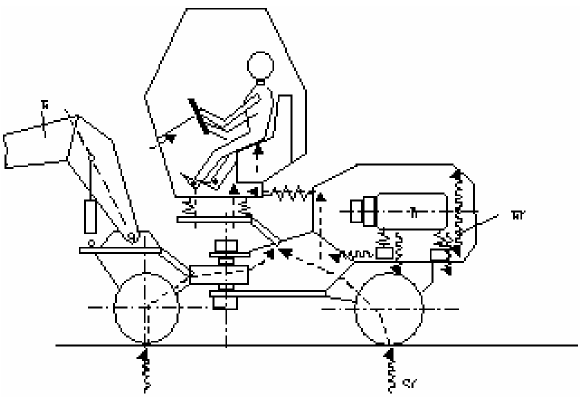

Figure 1. Principle scheme in case of an excavator having the cab elastic connected to the chassis and the excitation sources for the cab are the engine and the railway

1.2 Dynamic model in case of a construction machine using vibrations as determining factor during the working process (vibrating rollers, plates, vibrating rammers, asphalt finishers, pilling equipment)

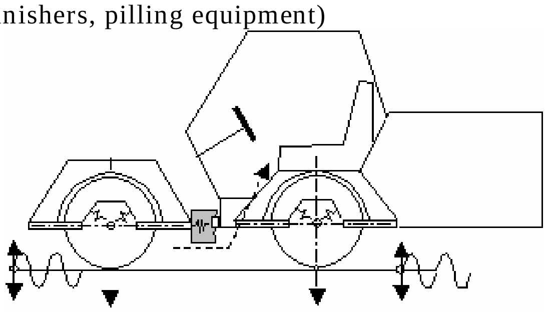

Figure 2. Self-propelled vibrating roller (two vibrating rollers)

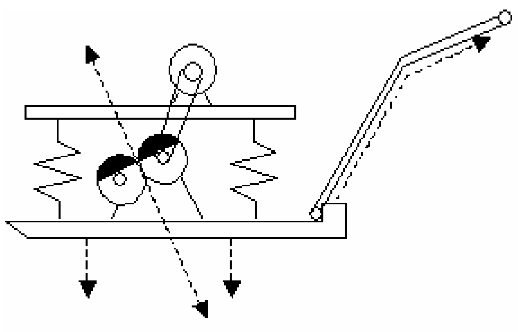

Figure 3. Vibrating compacting plate

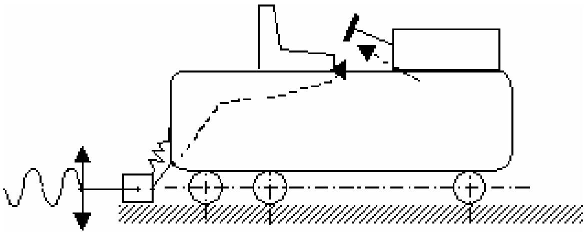

Figure 4. Asphalt finisher

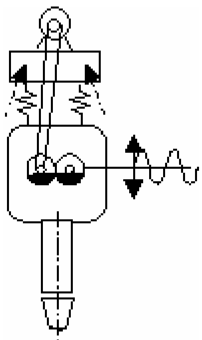

Figure 5. Pilling equipment

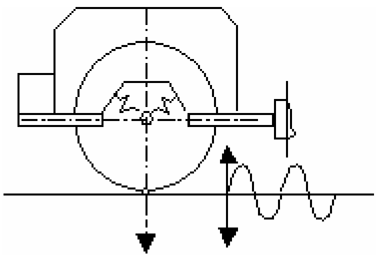

Figure 6. Towed vibrating roller

2. Dynamic Compatibility Conditions Concerning Operator’s Position-Cab System

The compatibility requirements are obtained as a result of the interaction between the cab and the elements of the operator’s position represented by chair, floor, controls, levers, and pedals. The cab is represented by the structural components consisting of welded rolled sections having as closing elements peripheral welded steel sheet and peripheral windows elastic attached by means of rubber sealant. So the envelope can be elastic insulated against the chassis or rigidly fixed onto this. Under the last condition according to the excitations given by the external vibrations the cab has to meet the operational requirements imposed by the environment inside the cab as well as to respect the permissible limits concerning the vibrations and noise at the operator’s position.

In order to attain both the controls operational level and a corresponding level for the transmitted vibrations the dynamic compatibility requirements are the following:

§ the insulation degree for the transmitted vibrations has to be at least 80%;

§ the staticdeflection achieved by the elastic system must ensure the cab insulation for three different cases obtained as a result of the following excitations:

§ low frequency vibrations (3 to 8 Hz) generated by the rolling track;

§ technological vibrations within the range 15 to 60 Hz;

§ interference vibrations generated by the vital machine parts (engines, pumps, control and power equipment) at a frequency of 20 to 40 Hz.

§ the reduction of the global amplification degree (for all possible vibration modes) by vibration modes decoupling. This requirement can be met during the design stage, when static decoupling is imposed by means of elastic elements and the dynamic decoupling by the mass elements;

§ avoiding of the dynamic operation zones in the vecinity of the resonance specific to cab-chassis system configuration

3. Analyze Of The Dynamic Behavior For The Elastic Connection Between The Cab And The Carrier

The differential motion equations, as a result of application of the second order Lagrange equation are under the form:

$$A\ddot{q} + Cq = f \tag{1}$$

where

A is the inertia matrix:

$$A = \begin{bmatrix} m & 0 & 0 & 0 & 0 & 0 \\ 0 & m & 0 & 0 & 0 & 0 \\ 0 & 0 & m & 0 & 0 & 0 \\ 0 & 0 & 0 & J_x & 0 & 0 \\ 0 & 0 & 0 & 0 & J_y & 0 \\ 0 & 0 & 0 & 0 & 0 & J_z \end{bmatrix}$$

C is the rigidity matrix:

$$C = \begin{bmatrix} \sum k_{ix} & 0 & 0 & 0 & \sum k_{ix} z_i & -\sum k_{ix} y_i \\ 0 & \sum k_{iy} & 0 & -\sum k_{iy} z_i & 0 & \sum k_{iy} x_i \\ 0 & 0 & \sum k_{iz} & \sum k_{iz} y_i & -\sum k_{iz} x_i & 0 \\ 0 & -\sum k_{iy} z_i & \sum k_{iz} y_i & \sum \left(k_{iy} z_i^2 + k_{iz} y_i^2\right) & -\sum k_{iz} x_i y_i & -\sum k_{iy} z_i x_i \\ \sum k_{ix} z_i & 0 & k_{iz} x_i & -\sum k_{iz} x_i y_i & \sum \left(k_{iz} x_i^2 + k_{ix} z_i^2\right) & -\sum k_{ix} y_i z_i \\ -\sum k_{ix} y_i & \sum k_{iy} x_i & 0 & -\sum k_{iy} z_i x_i & -\sum k_{ix} y_i z_i & \sum \left(k_{ix} y_i^2 + k_{iy} x_i^2\right) \end{bmatrix}$$

q is the generalized coordinates vector:

$$q = [q_1 \quad q_2 \quad q_3 \quad q_4 \quad q_5 \quad q_6]^T$$
$$= [X \quad Y \quad Z \quad \varphi_x \quad \varphi_y \quad \varphi_z]^T$$

$\dot{q}$ is the generalized speed vector:

$$\dot{q} = \begin{bmatrix} \dot{q}_1 & \dot{q}_2 & \dot{q}_3 & \dot{q}_4 & \dot{q}_5 & \dot{q}_6 \end{bmatrix}^T = \begin{bmatrix} \dot{X} & \dot{Y} & \dot{Z} & \dot{\varphi}_x & \dot{\varphi}_y & \dot{\varphi}_z \end{bmatrix}^T$$

$\ddot{q}$ is the generalized acceleration vector:

$$\ddot{q} = \begin{bmatrix} \ddot{q}_1 & \ddot{q}_2 & \ddot{q}_3 & \ddot{q}_4 & \ddot{q}_5 & \ddot{q}_6 \end{bmatrix}^T = \begin{bmatrix} \ddot{X} & \ddot{Y} & \ddot{Z} & \ddot{\varphi}_x & \ddot{\varphi}_y & \ddot{\varphi}_z \end{bmatrix}^T$$

f is the generalized perturbing force vector:

$$f = \begin{Bmatrix} f_1 \\ f_2 \\ f_3 \\ f_4 \\ f_5 \\ f_6 \end{Bmatrix} = \begin{Bmatrix} Q_1^F \\ Q_2^F \\ Q_3^F \\ Q_4^F \\ Q_5^F \\ Q_6^F \end{Bmatrix} = \begin{Bmatrix} Q_X^F \\ Q_Y^F \\ Q_Z^F \\ Q_{\varphi_x}^F \\ Q_{\varphi_y}^F \\ Q_{\varphi_z}^F \end{Bmatrix}$$

$$= \begin{Bmatrix} \sum_{k=1}^{p} F_{kx} \\ \sum_{k=1}^{p} F_{ky} \\ \sum_{k=1}^{p} F_{kz} \\ \sum_{k=1}^{p} \left(y_k F_{kz} - z_k F_{ky}\right) + \sum_{i=1}^{q} M_{ix} \\ \sum_{k=1}^{p} \left(z_k F_{kx} - x_k F_{kz}\right) + \sum_{i=1}^{q} M_{iy} \\ \sum_{k=1}^{p} \left(x_k F_{ky} - y_k F_{kx}\right) + \sum_{i=1}^{q} Mz \end{Bmatrix}$$

3.1 Vibration for one symmetry plan cab

3.1.1 Calculus model for one symmetry plan cab

The cab assumed to be underside elastic supported in four points has a longitudinal-vertical symmetry plan yCz (see figure 7).

The symmetry refers to:

- mass distribution;
- dimensions;
- connections (elastic constants, positions); the elastic supports are identical.

Due to the symmetry some of the rigidity matrix coupling terms become zero as follows:

$$\sum k_{iy} = 0; \quad \sum k_{iz} = 0; \quad \sum k_{iz} x_i y_i = 0; \quad \sum k_{iy} z_i x_i = 0 \qquad (2)$$

and the rigidity matrix results in:

$$C = \begin{bmatrix} 4k_x & 0 & 0 & 0 & -4hk_x & -2k_x(b_3-b_2) \\ 0 & 4k_y & 0 & 44hk_y & 0 & 0 \\ 0 & 0 & 4k_z & 2k_z(b_3-b_2) & 0 & 0 \\ 0 & 4hk_y & 2k_z(b_3-b_2) & 2\left[k_z\left(b_2^2+b_3^2\right)\right] & 0 & 0 \\ -4hk_x & 0 & 0 & 0 & 4\left(h^2k_x+a^2k_z\right) & 2hk_z(b_3-b_2) \\ -2k_z(b_3-b_2) & 0 & 0 & 0 & 2hk_z(b_3-b_2) & 2\left[2a^2k_y+k_x\left(b_2^2+b_3^2\right)\right] \end{bmatrix} \qquad (3)$$

The tabel form for the rigidity matrix is:

$$C = \left[\begin{array}{cccccc|c} X & Y & Z & \varphi_x & \varphi_y & \varphi_z & \\ 4k_x & 0 & 0 & 0 & -4hk_x & -2k_x(b_3-b_2) & X \\ 0 & 4k_y & 0 & 4hk_y & 0 & 0 & Y \\ 0 & 0 & 4k_z & 2k_z(b_3-b_2) & 0 & 0 & Z \\ 0 & 4hk_y & 2k_z(b_3-b_2) & 2\left[k_z\left(b_2^2+b_3^2\right)+2h^2k_y\right] & 0 & 0 & \varphi_x \\ -4hk_x & 0 & 0 & 0 & 4\left(h^2k_x+a^2k_z\right) & 2hk_x(b_3-b_2) & \varphi_y \\ -2k_x(b_3-b_2) & 0 & 0 & 0 & 2hk_x(b_3-b_2) & 2\left[2a^2k_y+k_x\left(b_2^2+b_3^2\right)\right] & \varphi_z \end{array}\right]$$

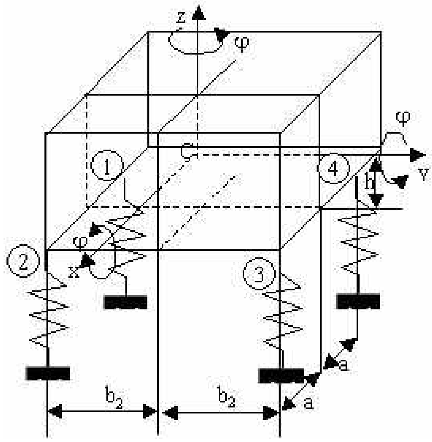

Figura 7 One longitudinal-vertical symmetry plan rigid body four points elastic supported

One can note that, nullifying coupling terms (2), the system decouples into two sub-systems described by coordinates (Y, Z, φ_x) and (X, φ_y, φ_z), and their matrix are:

- for the (Y, Z, φ_x) sub-system:

$$A_3 = \begin{bmatrix} m & 0 & 0 \\ 0 & m & 0 \\ 0 & 0 & m \end{bmatrix} \text{ inertia matrix} \qquad (4)$$

$$C_3 = \begin{bmatrix} 4k_y & 0 & 4hk_y \\ 0 & 4k_z & 2k_z(b_3-b_2) \\ 4hk_y & 2k_z(b_3-b_2) & 2\left[k_z\left(b_2^2+b_3^2\right)+2h^2k_y\right] \end{bmatrix} \qquad (5)$$

rigidity matrix

- for the (X, φ_y, φ_z) sub-system:

$$A_4 = \begin{bmatrix} m & 0 & 0 \\ 0 & J_y & 0 \\ 0 & 0 & J_z \end{bmatrix} \text{ inertia matrix} \qquad (6)$$

$$C_4 = \begin{bmatrix} 4k_x & -4hk_x & -2hk_x(b_3-b_2) \\ -4hk_x & 4(h^2k_x + a^2k_z) & 2hk_x(b_3-b_2) \\ -2k_x(b_3-b_2) & 2hk_x(b_3-b_2) & 2[2a^2k_y + k_x(b_2^2+b_3^2)] \end{bmatrix} \quad (7)$$

rigidity matrix

3.1.2 Free vibrations analyze in case of one symmetry cab elastic supported

3.1.2.1 Motion equations

Taking into account the two systems decoupling, the differential equations in case of free vibrations result in:

- for coupled longitudinal, vertical and ptching vibrations:

$$\begin{cases} m\ddot{Y} + 4k_yY + 4hk_y\varphi_x = 0 \\ m\ddot{Z} + 4k_zZ + 2k_z(b_3-b_2)\varphi_x = 0 \\ J_x\ddot{\varphi}_x + 4hk_yY + 2k_z(b_3-b_2)Z + 2[k_z(b_2^2+b_3^2) + 2h^2k_y]\varphi = 0 \end{cases} \quad (8)$$

- for coupled lateral, rolling and gyration vibrations:

$$\begin{cases} m\ddot{X} + 4k_xX - 4hk_x\varphi_y - 2k_x(b_3-b_2)\varphi_z = 0 \\ J_y\ddot{\varphi}_y - 4hk_xX + 4(h^2k_x + a^2k_z)\varphi_y + 2hk_x(b_3-b_2)\varphi_z = 0 \\ J_z\ddot{\varphi}_z - 2k_x(b_3-b_2)X + 2k_x(b_3-b_2)\varphi_y + 2[2a^2k_y + k_x(b_2^2+b_3^2)]\varphi_z = 0 \end{cases} \quad (9)$$

3.1.2.2 Eigen pulsations. Eigen vectors

a) the (Y, Z, φ_x) sub-system

The eigen pulsation equation is under the form:

$$\begin{aligned} &(4k_y - p^2m)(4k_z - p^2m)\{2[k_z(b_2^2+b_3^2) + 2h^2k_y] - p^2J_x\} - \\ &-[2k_z(b_3-b_2)]^2(4k_y - p^2m) - (4hk_y)^2(4k_z - p^2m) = 0 \end{aligned} \quad (10)$$

Having in regard the eigen pulsation equations of all six uncoupled motions:

$$p_X = \sqrt{\frac{\sum k_{ix}}{m}} = 2\sqrt{\frac{k_x}{m}} \quad (11)$$

$$p_Y = \sqrt{\frac{\sum k_{iy}}{m}} = 2\sqrt{\frac{k_y}{m}} \quad (12)$$

$$p_Z = \sqrt{\frac{\sum k_{iz}}{m}} = 2\sqrt{\frac{k_z}{m}} \quad (13)$$

$$p_{\varphi_x} = \sqrt{\frac{\sum y_i^2k_{iz} + \sum z_i^2k_{iy}}{J_x}} = \sqrt{\frac{2[(b_2^2+b_3^2)k_z + 2h^2k_y]}{J_x}} \quad (14)$$

$$p_{\varphi_y} = \sqrt{\frac{\sum z_i^2k_{ix} + \sum x_i^2k_{iz}}{J_y}} = \sqrt{\frac{2h^2k_x + a^2k_z}{J_y}} \quad (15)$$

$$p_{\varphi_z} = \sqrt{\frac{\sum x_i^2k_{iy} + \sum y_i^2k_{ix}}{J_z}} = \sqrt{\frac{2[2a^2k_y + (b_2^2+b_3^2)k_x]}{J_z}} \quad (16)$$

the eigen pulsation equation can be rewritten as:

$$\begin{aligned} &(p_Y^2 - p^2)(p_Z^2 - p^2)(p_{\varphi_x}^2 - p^2) - \\ &-(p_Y^2 - p^2)\alpha_1\alpha_2 - (p_Z^2 - p^2)\alpha_3\alpha_4 = 0 \end{aligned} \quad (17)$$

with the coupling factors:

$$\alpha_1 = \frac{1}{m}\sum k_{iy}z_i = -\frac{4}{m}hk_y \quad (18)$$

$$\alpha_2 = \frac{1}{J_x}\sum k_{iy}z_i = -\frac{4}{J_x}hk_y \quad (19)$$

$$\alpha_3 = \frac{1}{m}\sum k_{iz}y_i = \frac{2}{m}[b_3 - b_2]k_z \quad (20)$$

$$\alpha_4 = \frac{1}{J_x}\sum k_{iy}z_i = \frac{2}{J_x}(b_3 - b_2)k_z \quad (21)$$

The eigen pulsation equation under the polynomial formis:

$$p^6 - \Omega_1p^4 + \Omega_2p^2 - \Omega_3 = 0 \quad (22)$$

where

$$\Omega_1 = p_Y^2 + p_Z^2 + p_{\varphi_x}^2 \quad (23)$$

$$\Omega_2 = p_Y^2p_Z^2 + p_Z^2p_{\varphi_x}^2 + p_{\varphi_x}^2p_Y^2 - \alpha_1\alpha_2 - \alpha_3\alpha_4 \quad (24)$$

$$\Omega_2 = p_Y^2p_Z^2p_{\varphi_x}^2 - p_Y^2\alpha_1\alpha_2 - p_Z^2\alpha_3\alpha_4 \quad (25)$$

Changing the variable:

$$p^2 = w + \frac{\Omega_1}{3} \quad (26)$$

equation 22 becomes:

$$w^3 - 3s_1w - 2q_1 = 0 \quad (27)$$

with the notations:

$$s_1 = \left(\frac{\Omega_1}{3}\right)^2 - \frac{\Omega_2}{3} \quad (28)$$

$$q_1 = \left(\frac{\Omega_1}{3}\right)^2 - \frac{\Omega_1\Omega_2}{6} + \frac{\Omega_3}{2} \quad (29)$$

The equation (27) solutions are the following:

$$w_1 = \pm 2\sqrt{s_1}\cos\frac{\gamma_1}{3} \qquad (30)$$

$$w_2 = \pm 2\sqrt{s_1}\cos\frac{\pi - \gamma_1}{3} \qquad (31)$$

$$w_3 = \pm 2\sqrt{s_1}\cos\frac{\pi + \gamma_1}{3} \qquad (32)$$

where

$$\gamma_1 = \arccos\frac{|q_1|}{\sqrt{s_1^3}} \qquad (33)$$

Sign “+” corresponds to the case $q_1 > 0$, and sign “-“ to the case $q_1 < 0$.
After calculating the equation roots in w, the three squared eigen pulsations are determined basing on the formula:

$$p_i^2 = w_i + \frac{\Omega_1}{3} \quad i = \overline{1,3} \qquad (34)$$

In case one solves the equation (27) using an iterative method, the recommende values for the initial iterations are the following:

$$w_1^{(0)} = 0\,;\; w_2^{(0)} = -2\sqrt{s_1}\,;\; w_3^{(0)} = 2\sqrt{s_1} \qquad (35)$$

b) (X, φ_y, φ_z) sub-system
The eigen pulsation equation is:

$$\begin{aligned}&\left(4k_x - p^2 m\right)\left[4\left(h^2 k_x + a^2 k_z\right) - p^2 J_y\right]\\&\left\{2\left[2a^2 k_y + k_x\left(b_2^2 + b_3^2\right)\right] - p^2 J_z\right\} +\\&+ 2\left(4hk_x\right)\left[2hk_x\left(b_3 - b_2\right)\right] -\\&-\left[2hk_x\left(b_3 - b_2\right)\right]^2\left(4k_x - p^2 m\right) -\\&-\left[-2k_x\left(b_3 - b_2\right)\right]^2\left[4\left(h^2 k_x + a^2 k_z\right) - p^2 J_y\right] -\\&-\left(-4hk_x\right)^2\left\{2\left[2a^2 k_y + k_x\left(b_2^2 + b_3^2\right)\right] - p^2 J_z\right\} = 0\end{aligned} \qquad (36)$$

Let note the coupling terms:

$$\beta_1 = \frac{1}{m}\sum k_{ix} z_i = -\frac{4}{m} hk_x \qquad (37)$$

$$\beta_2 = \frac{1}{J_y}\sum k_{ix} z_i = -\frac{4}{J_y} hk_x \qquad (38)$$

$$\beta_3 = \frac{1}{m}\sum k_{ix} y_i = \frac{2}{m} k_x\left(b_3 - b_2\right) \qquad (39)$$

$$\beta_4 = \frac{1}{J_z}\sum k_{ix} y_i = \frac{2}{J_z} k_x\left(b_3 - b_2\right) \qquad (40)$$

$$\beta_5 = \frac{1}{J_y}\sum k_{ix} y_i z_i = -\frac{2}{J_y} hk_x\left(b_3 - b_2\right) \qquad (41)$$

$$\beta_6 = \frac{1}{J_z}\sum k_{ix} y_i z_i = -\frac{2}{J_z} hk_x\left(b_3 - b_2\right) \qquad (42)$$

and the eigen pulsation equation becomes:

$$\begin{aligned}&\left(p_X^2 - p^2\right)\left(p_{\varphi_y}^2 - p^2\right)\left(p_{\varphi_x}^2 - p^2\right) +\\&+ 2\beta_2\beta_3\beta_6 - \beta_5\beta_6\left(p_X^2 - p^2\right) -\\&-\beta_3\beta_4\left(p_{\varphi_y}^2 - p^2\right) - \beta_1\beta_2\left(p_{\varphi_x}^2 - p^2\right) = 0\end{aligned} \qquad (43)$$

After grouping the terms depending on the powers of p^6, it results in:

$$p^6 - \Psi_1 p^4 + \Psi_2 p^2 - \Psi_3 = 0 \qquad (44)$$

where

$$\Psi_1 = p_Y^2 + p_Z^2 + p_{\varphi_x}^2 \qquad (45)$$

$$\begin{aligned}\Psi_2 &= p_Y^2 p_Z^2 + p_Z^2 p_{\varphi_x}^2 + p_{\varphi_x}^2 p_Y^2 -\\&- \beta_1\beta_2 - \beta_3\beta_4 - \beta_5\beta_6\end{aligned} \qquad (46)$$

$$\begin{aligned}\Psi_3 &= p_Y^2 p_Z^2 p_{\varphi_x}^2 - p_Y^2 \beta_5\beta_6 -\\&- p_{\varphi_y}^2 \beta_3\beta_4 - p_{\varphi_z}^2 \beta_1\beta_2 + 2\beta_2\beta_3\beta_6\end{aligned} \qquad (47)$$

Making the substitution

$$p^2 = u + \frac{w_1}{3} \qquad (48)$$

and notations

$$s_2 = \left(\frac{\Psi_1}{3}\right)^2 - \frac{\Psi_2}{3} \qquad (49)$$

$$q_2 = \left(\frac{\Psi_1}{3}\right)^3 - \frac{\Psi_1\Psi_2}{6} + \frac{\Psi_3}{3} \qquad (50)$$

the equation (44) solutions are obtained under the form:

$$u_1 = \pm 2\sqrt{s_2}\cos\frac{\gamma_2}{3} \qquad (51)$$

$$u_2 = \pm 2\sqrt{s_2}\cos\frac{\pi - \gamma_2}{3} \qquad (52)$$

$$u_3 = \pm 2\sqrt{s_2}\cos\frac{\pi + \gamma_2}{3} \qquad (53)$$

with

$$\gamma_2 = \arccos\frac{|q_2|}{\sqrt{s_2^3}} \qquad (54)$$

Sign “+” corresponds to the case $q_2 > 0$, and sign “-“ to the case $q_2 < 0$.
The squared eigen pulsations are:

$$p_i^2 = u_{i-3} + \frac{\Psi_1}{3} \qquad i = \overline{4,6} \quad (55)$$

Aiming iterative numerical equation solving the following initial iterations are recommended:

$$u_1^{(0)} = 0; \qquad u_2^{(0)} = -2\sqrt{s_2}; \qquad u_3^{(0)} = 2\sqrt{s_2} \quad (56)$$

The eigen vectors in case of the six eigen vibration modes are:

- for the (Y, Z, φ_x) sub-system

$$\upsilon_i = \begin{Bmatrix} 1 \\ \mu_{2i} \\ \mu_{3i} \end{Bmatrix} \; i = \overline{1,3} \quad (57)$$

where the eigen values are determined by relations:

$$\begin{cases} \mu_{2i} = -\dfrac{\alpha_1\alpha_3}{\left(p_Y^2 - p_i^2\right)\left(p_Z^2 - p_i^2\right)} \\ \mu_{3i} = \dfrac{\alpha_1}{p_Y^2 - p_i^2} \end{cases} \quad i = \overline{1,3} \quad (58)$$

- for the (X, φ_y, φ_z) sub-system

$$\upsilon_i = \begin{Bmatrix} 1 \\ \mu_{2i} \\ \mu_{3i} \end{Bmatrix} \; i = \overline{4,6} \quad (59)$$

with the eigen values:

$$\begin{cases} \mu_{2i} = \dfrac{\beta_5\left(p_X^2 - p_i^2\right) - \beta_2\beta_3}{\beta_3\left(p_{\varphi_y}^2 - p_i^2\right) - \beta_1\beta_5} \\ \mu_{3i} = \dfrac{\left(p_X^2 - p_i^2\right)\left(p_{\varphi_y}^2 - p_i^2\right) + \beta_1\beta_2}{\beta_3\left(p_{\varphi_y}^2 - p_i^2\right) - \beta_1\beta_5} \end{cases} \quad (60)$$

3.1.2.3 Motion laws in case of free vibrations

In case of free vibrations, the motion laws are given by the following expressions:

$$\begin{cases} Y(t) = C_1\sin\left(p_1 t + \theta_1\right) + \\ + C_2\sin\left(p_2 t + \theta_2\right) + C_3\sin\left(p_3 t + \theta_3\right) \\ Z(t) = \mu_{21}C_1\sin\left(p_1 t + \theta_1\right) + \\ + \mu_{22}C_2\sin\left(p_2 t + \theta_2\right) + \mu_{23}C_3\sin\left(p_3 t + \theta_3\right) \\ \varphi_x(t) = \mu_{31}C_1\sin\left(p_1 t + \theta_1\right) + \\ + \mu_{32}C_2\sin\left(p_2 t + \theta_2\right) + \mu_{33}C_3\sin\left(p_3 t + \theta_3\right) \end{cases} \quad (61)$$

and

$$\begin{cases} X(t) = C_4\sin\left(p_4 t + \theta_4\right) + \\ + C_5\sin\left(p_5 t + \theta_2\right) + C_6\sin\left(p_6 t + \theta_6\right) \\ \varphi_y(t) = \mu_{24}C_4\sin\left(p_4 t + \theta_4\right) + \\ + \mu_{25}C_5\sin\left(p_5 t + \theta_5\right) + \mu_{26}C_6\sin\left(p_6 t + \theta_6\right) \\ \varphi_z(t) = \mu_{34}C_4\sin\left(p_4 t + \theta_4\right) + \\ + \mu_{35}C_5\sin\left(p_5 t + \theta_5\right) + \mu_{36}C_3\sin\left(p_6 t + \theta_6\right) \end{cases} \quad (62)$$

where

μ_{ji} (j = 2,3; i = 1, 2, .., 6) are the eigen values

C_1, .., C_6, θ_1, .., θ_6 are integration constants and their values are determined from the initial conditions (displacement and speed):

$$\begin{aligned} & X(0) = X_0;\; Y(0) = Y_0; \\ & Z(0) = Z_0;\; \varphi_x(0) = \varphi_{x0}; \\ & \varphi_y(0) = \varphi_{y_0};\; \varphi_z(0) = \varphi_{z0}; \\ & \dot{X}(0) = \dot{X}_0;\; \dot{Y}(0) = \dot{Y}_0; \\ & \dot{Z}(0) = \dot{Z}_0;\; \dot{\varphi}_x(0) = \dot{\varphi}_{x0}; \\ & \dot{\varphi}_y(0) = \dot{\varphi}_{y_0};\; \dot{\varphi}_z(0) = \dot{\varphi}_{z0} \end{aligned} \quad (63)$$

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