THE OPERATIONAL SPACE'S MICRO-GEOMETRICAL ANALYSIS OF THE R⊥R||R KINEMATICS CHAIN

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ABSTRACT

In order to systematize the three monomobile coupling kinematic chains was essential to find criteria which eliminate those coupling which are inefficient, from the mobility point of view. It is embraced, as main criteria, the robot's manageability, through the volume element analysis, removing, this way, the decreased mobility points from the working space.

KEYWORDS: kinematics, chains, operational space, monomobile coupling.

1. Introduction

We are considering the kinematics chains which consist of only R and T monomobile coupling, whose axis may have the relative positions: ≡, ||, ⊥, +.

There were identified the following variants: R⊥R⊥R, R||R⊥R, R⊥R||R, R⊥R⊥T, T ⊥R⊥R, R⊥R||T, T||R⊥R, R||R||T, T||R||R, R⊥T⊥ R⊥, R⊥T||R, R||T⊥R, R||T||R, T⊥R||T, T||R⊥T, T⊥T||R, R||T⊥T, T⊥T⊥R||, R⊥T⊥T||, T⊥T⊥T⊥.

2. Description method of analysis

The above reviewed structural variants have no degenerate operational spaces (the number of independent parameters associated with the operational space equals the positioning of the kinematics chain degree of liberty).

For the open kinematics chain with three degrees of liberty we consider that the volume element is the main local qualitative assessment criterion of manageability within the operational space.

Assuming that the absolute value of the three vectors mixed product, geometrically speaking, represents the parallelipiped's volume built on the three vectors, we are going to define the operational space volume element as the vectors mixed product

$$\bar{r}'_{q1} dq_1 = \frac{\partial \bar{r}}{\partial q_1} dq_1;$$

$$\bar{r}'_{q2} dq_2 = \frac{\partial \bar{r}}{\partial q_2} dq_2; \qquad \bar{r}'_{q3} dq_3 = \frac{\partial \bar{r}}{\partial q_3} dq_3, \tag{1}$$

calculated in the point M, located at the intersection of the coordinated curves Γ_{q1}, Γ_{q2} and Γ_{q3} (Figure 1).

Taking into consideration the three vectors mixed product definition, the final expression for volume element will be:

$$dV = \begin{vmatrix} \frac{\partial \rho_M^x}{\partial q_1} & \frac{\partial \rho_M^y}{\partial q_1} & \frac{\partial \rho_M^z}{\partial q_1} \\ \frac{\partial \rho_M^x}{\partial q_2} & \frac{\partial \rho_M^y}{\partial q_2} & \frac{\partial \rho_M^z}{\partial q_2} \\ \frac{\partial \rho_M^x}{\partial q_3} & \frac{\partial \rho_M^y}{\partial q_3} & \frac{\partial \rho_M^z}{\partial q_3} \end{vmatrix} dq_1 dq_2 dq_3 =$$

$$= \left[\frac{\partial \rho_M^x}{\partial q_1} \left(\frac{\partial \rho_M^y}{\partial q_2} \frac{\partial \rho_M^z}{\partial q_3} - \frac{\partial \rho_M^z}{\partial q_2} \frac{\partial \rho_M^y}{\partial q_3} \right) + \right.$$
$$+ \frac{\partial \rho_M^y}{\partial q_1} \left(\frac{\partial \rho_M^z}{\partial q_2} \frac{\partial \rho_M^x}{\partial q_3} - \frac{\partial \rho_M^x}{\partial q_2} \frac{\partial \rho_M^z}{\partial q_3} \right) +$$
$$\left. + \frac{\partial \rho_M^z}{\partial q_1} \left(\frac{\partial \rho_M^x}{\partial q_2} \frac{\partial \rho_M^y}{\partial q_3} - \frac{\partial \rho_M^y}{\partial q_2} \frac{\partial \rho_M^x}{\partial q_3} \right) \right] dq_1 dq_2 dq_3 . \tag{2}$$

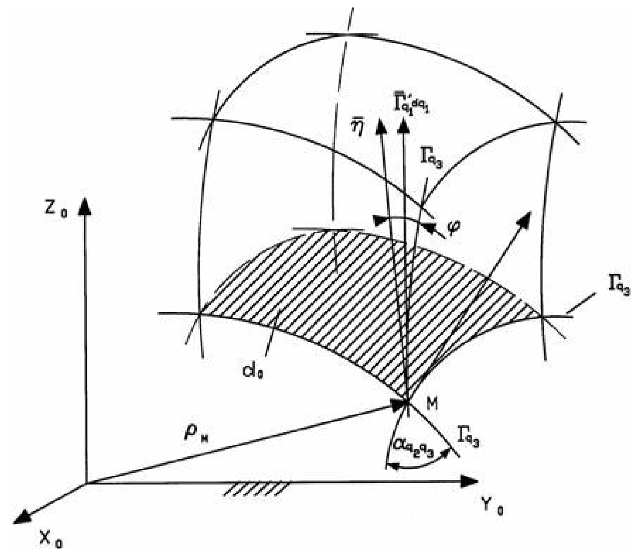

Figure 1. Operational space volume element

The φ angle can be computed using an expression like:

$$\cos\varphi = \frac{A_1A_2 + B_1B_2 + C_1C_2}{\sqrt{A_1^2 + B_1^2 + C_1^2}\sqrt{A_2^2 + B_2^2 + C_2^2}}, \qquad (3)$$

where:

$$\frac{\partial \bar{\rho}_M}{\partial q_1} = A_1\bar{i} + B_1\bar{j} + C_1\bar{k}; \qquad (4)$$

$$\left(\frac{\partial \bar{\rho}_M}{\partial q_2} \times \frac{\partial \bar{\rho}_M}{\partial q_3}\right) = A_2\bar{i} + B_2\bar{j} + C_2\bar{k} \qquad (5)$$

We will define, as a single point, within the operational space of the tri-mobile positioning kinematic chain, the point where dV = 0. If dV = 0, the characteristic point M manageability decreases, in short M will move in the vicinity of a single point either on a regular surface or on a curve, for infinitesimal displacements at the kinematic chain couples. It’s important to emphasize that the surrounding M point volume doesn’t have any discontinuity.

We consider the kinematics positioning chain R⊥R||R, showed in Figure 2.

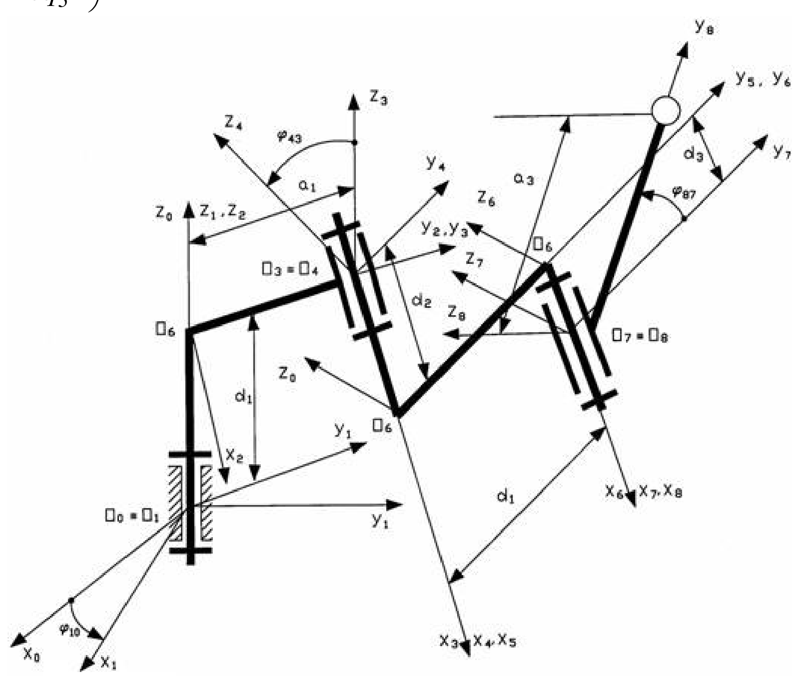

Figure 2. Kinematics positioning chain R⊥R||R

The characteristic M point position, as against the fixed reference system, is given by the expression:

$$
\begin{aligned}
(\rho_M)=&(r_{10})+[a_{10}]^T(r_{21})+[a_{10}]^T[a_{21}]^T(r_{32})+\\
&+[a_{10}]^T[a_{21}]^T[a_{32}]^T(r_{43})+\\
&+[a_{10}]^T[a_{21}]^T[a_{32}]^T[a_{43}]^T(r_{54})+\\
&+[a_{10}]^T[a_{21}]^T[a_{32}]^T[a_{43}]^T[a_{54}]^T(r_{65})+\\
&+[a_{10}]^T[a_{21}]^T[a_{32}]^T[a_{43}]^T[a_{54}]^T[a_{65}]^T(r_{76})+\\
&+[a_{10}]^T[a_{21}]^T[a_{32}]^T[a_{43}]^T[a_{54}]^T[a_{65}]^T[a_{76}]^T(r_{87})+\\
&+[a_{10}]^T[a_{21}]^T[a_{32}]^T[a_{43}]^T[a_{54}]^T[a_{65}]^T[a_{76}]^T[a_{87}]^T(r_M).
\end{aligned} \tag{6}
$$

where

$$
\begin{aligned}
&(r_{10})=(0);\ (r_{43})=(0);\ (r_{87})=(0);\\
&[a_{21}]^T=[E];[a_{32}]^T=[E];\\
&[a_{54}]^T=[E];\ [a_{65}]^T=[E];\\
&[a_{76}]=[E].\\
&(r_{21})^T=[0\quad 0\quad d_1];\\
&(r_{32})^T=[0\quad a_1\quad 0];\\
&(r_{54})^T=[d_2\quad 0\quad 0];\\
&(r_{65})^T=[0\quad a_2\quad 0];\\
&(r_{76})^T=[d_3\quad 0\quad 0];\\
&(r_M)^T=[0\quad a_3\quad 0];\\
&[a_{10}]^T=\begin{bmatrix}\cos\varphi_{10} & -\sin\varphi_{10} & 0\\ \sin\varphi_{10} & \cos\varphi_{10} & 0\\ 0 & 0 & 1\end{bmatrix};\\
&[a_{43}]^T=\begin{bmatrix}1 & 0 & 0\\ 0 & \cos\varphi_{43} & -\sin\varphi_{43}\\ 0 & \sin\varphi_{43} & \cos\varphi_{43}\end{bmatrix};\\
&[a_{87}]^T=\begin{bmatrix}1 & 0 & 0\\ 0 & \cos\varphi_{87} & -\sin\varphi_{87}\\ 0 & \sin\varphi_{87} & \cos\varphi_{87}\end{bmatrix}.
\end{aligned} \tag{7}
$$

Replacing (7) in (6) and computing, we obtain:

$$
\begin{aligned}
\rho_M^x =& -a_1\sin\varphi_{10}+(d_2+d_3)\cos\varphi_{10}-\\
&-a_2\sin\varphi_{10}\cos\varphi_{43}-\\
&-a_3\sin\varphi_{10}\cos(\varphi_{43}+\varphi_{87});
\end{aligned}
$$

$$
\begin{aligned}
\rho_M^y =& a_1\cos\varphi_{10}+(d_2+d_3)\sin\varphi_{10}+\\
&+a_2\cos\varphi_{10}\cos\varphi_{43}+\\
&+a_3\cos\varphi_{10}\cos(\varphi_{43}+\varphi_{87});
\end{aligned}
$$

$$
\rho_M^z = d_1+a_2\sin\varphi_{43}+a_3\sin(\varphi_{43}+\varphi_{87}).
$$

Using the following notations

$$
\begin{aligned}
&\sin\varphi_{10}\equiv x_1;\cos\varphi_{10}\equiv x_2\Rightarrow x_1^2+y_1^2=1;\\
&\sin\varphi_{43}\equiv y_1;\cos\varphi_{43}\equiv y_2\Rightarrow y_1^2+y_1^2=1;\\
&\sin(\varphi_{43}+\varphi_{87})\equiv z_1;\\
&\cos(\varphi_{43}+\varphi_{87})\equiv z_2\Rightarrow z_1^2+z_2^2=1,
\end{aligned}
$$

the dV volume element expression, becomes:

$$
\begin{aligned}
&dV=\begin{vmatrix}a11 & a12 & a13\\ a21 & a22 & a23\\ a31 & a32 & a33\end{vmatrix}d\varphi_{10}d\varphi_{43}d\varphi_{87}\\
&a11=a_1x_2+(d_2+d_3)x_1+a_2x_2y_2+a_3x_2z_2\\
&a12=a_1x_1-(d_2+d_3)x_2+a_2x_1y_2+a_3x_1z_2\\
&a13=0\\
&a21=a_2x_1y_1+a_3x_1z_1\\
&a22=-a_2x_2y_1-a_3x_2z_1\\
&a23=a_2y_2+a_3z_2\\
&a31=a_3x_1z_1\\
&a32=-a_3x_2z_1\\
&a33=a_3z_2
\end{aligned} \tag{8}
$$

Enhancing the determinant against the first rows and doing the calculations will result the volume element expression as follows:

$$
dV=a_2a_3\left|A\sin\varphi_{87}\right|d\varphi_{10}d\varphi_{43}d\varphi_{87} \tag{9}
$$

where

$$
A=[a_1+a_2\cos\varphi_{43}+a_3\cos(\varphi_{43}+\varphi_{87})].
$$

For avoiding singularity, the condition is $|dV|\neq 0$, which means

$$
\sin\varphi_{87}\neq 0\Rightarrow \varphi_{87}\neq k\pi,\quad k\in Z \tag{10}
$$

or

$$
a_1+a_2\cos_{43}+a_3\cos(\varphi_{43}+\varphi_{87})\neq 0. \tag{11}
$$

On the basis of (3), (4) and (5) there will be:

$$cos\varphi = -\frac{A}{\sqrt{A^2 + (d_2 + d_3)^2}} \qquad (12)$$

Reviewing the 12th expression, there can be noticed that

$$\varphi = (2k+1)\,\pi \;\; (k \in Z), \quad \text{if} \quad d_2 + d_3 = 0.$$

3. Conclusions

The most favourable volume element has a parallelipiped shape, with the sides length approximately equal, because, in the vicinity of *M* point, the relative movements of kinematic elements will be, on an average, on a decreased amplitude and, therefore, the acting devices will provide small displacements.

Those conditions involve the big dimensions and small weights in comparison with other variants.

In the future implementation research, this study will be grounded in developing a computational program on the optimization of the industrial robots operational space.

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