A NEW CALCULATING METHOD OF THE REACTION SUPPORT GROUPS USED IN ROTARY KILN

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ABSTRACT

This paper presents a new analytical method for calculating the reaction of support rotary kiln, regarded as an indeterminate static problem. It is the Ψ-Method and equation of three displacements (using Ψ loading function) [3],[4], which eliminate the disadvantage of the classic graphical - analytical method CLAPEYRON. This method allows an easier calculation of reaction in situations when it is necessary to consider uneven supports as well.

KEYWORDS: rotary kilns, reaction of support, indeterminate systems

1. Overview

In order to calculate the resistance of support rotary kiln, the assumption that it is supported on a sufficient number, depending on the length of beam [1], [2] is necessary. For the rotary kiln, characterized by the high values L / D (length / diameter), it is necessary to use three or more support groups, depending on the length of systems. Hence, to reduce the investment, installation and maintenance costs, the rotary kiln is supported with a lower number support (usually three groups of support) [2]. In this situation, the rotary kiln may be regarded as a static indeterminate system (or hyper static system).
Support groups are designed, manufactured and installed so as all the support groups could be placed at the same level in relation to the reference axis [1], [2]. Because the mounting and adjustment or accidental causes generate errors, there is the possibility that at least one support span is not at the same level as the other (left column) as a consequence of an overload or a download the near support. In the calculations one must be aware of this situation.
It may be easier to determine the reaction of groups support the rotary kiln [3], [4] using the proposed *Ψ-Method* and the three-displacement equation .

2. Equation of three displacements

This is a new analytical method based on the equations of strains. It eliminates the disadvantages of graphical and analytical method CLAPEYRON. If we write the displacement using the loading function Ψ corresponding to a section of the bar, located at a distance x, we obtain:

$$E \cdot I \cdot w(x) = E \cdot I \cdot w_0 + E \cdot I \cdot \varphi_0 \cdot x + \Psi(x) \quad (1)$$

where:
w_0 is the displacement of the end section beam in *Oxz* system of coordinates
φ_0 – the slope of section corresponding to the end section beam;
$E \cdot I$ - flexural rigidity of beam, considered constant as far as its length is concerned;
$\Psi(x)$ – loading function.
If we consider the three sections of the bar between notated with *i, j, k* the corresponding displacements w_i, w_j respectively w_k (Fig. 1) are written:

$$\begin{cases} EI \cdot w_i = \Psi(x_i) + EI \cdot \varphi_0 \cdot x_i + EI \cdot w_0 \\ EI \cdot w_j = \Psi(x_j) + EI \cdot \varphi_0 \cdot (x_i + L_i) + EI \cdot w_0 \\ EI \cdot w_k = \Psi(x_k) + EI \cdot \varphi_0 \cdot (x_i + L_i) + EI \cdot w_0 \end{cases} \quad (2)$$

Multiplying the first of the equations (2) by L_j , the second by - (L_i+L_j) and the third by L_i , adding them to remove the unknowns w_0 and φ_0, we obtain the *equation of three displacements:*

$$EI\left[w_i L_j - w_j (L_j + L_i) + w_k L_i\right] = \\ = \Psi_i L_j - \Psi_j (L_j + L_i) + \Psi_k L_i \quad (3)$$

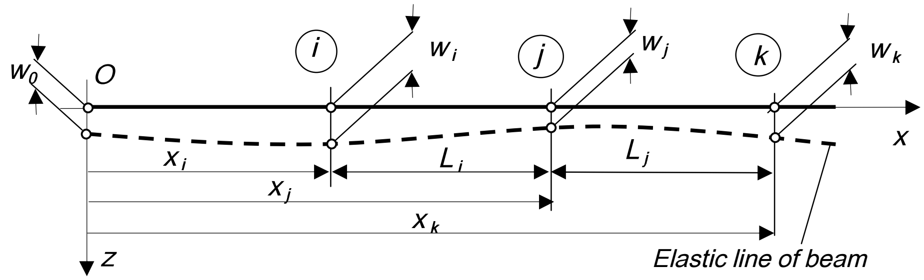

Fig. 1. The scheme for calculating the reaction

The loading functions Ψ_i , Ψ_j respectively Ψ_k are obtained by adding together the functions corresponding to all external loading (known) and reaction (unknown) of support beam.

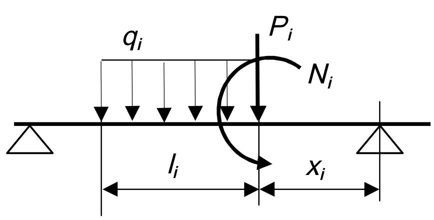

Fig. 2. The scheme for determining the functions of external load tasks.

The loading functions corresponding to section *i* depend on the nature of the load and are calculated with the relations (Fig. 2):

• P_i concentrated force acting at distances x_i of section *i:*

$$\Psi_s = \frac{P_i \cdot x_i^3}{6} \tag{4'}$$

• q_i uniformly distributed force on distance l_i, acting at distances x_i of section *i* :

$$\Psi_s = \frac{q_i \cdot (l_i + x_i)^4}{24} - \frac{q \cdot x_i^4}{24} \tag{4''}$$

• N_i concentrated couples acting at distances x_i of section *i* :

$$\Psi_s = \frac{N_i \cdot x_i^2}{2} \tag{4'''}$$

3. The beam with three uneven supports

We may consider the case of general beam with uneven support, loaded with all kinds of tasks outside (Fig. 3):

• P_i concentrated forces acting at distances d_i;

• q_i uniformly distributed loads acting on distance e_i-f_i;

• N_i concentrated couples acting at distances g_i.

There are assumptions of two cases:

a. The beam has flexural rigidity *EI* constant throughout its length and the supports are at the same level;

b. The beam keeps contact with the uneven supports (in Figure 3, support (2)), even if no outside task acts on it.

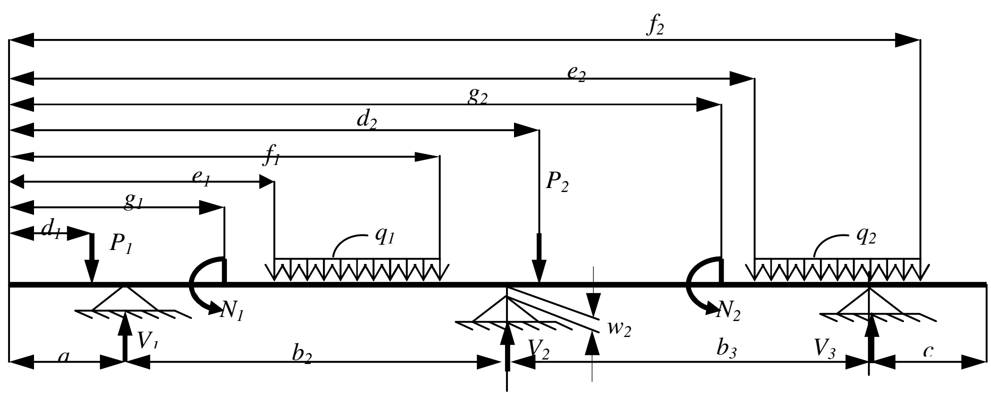

Fig. 3. Beam with three uneven supports

Considering that all three are uneven supports *(w_1, w_2, w_3)*, in order to determine the three reactions V_1, V_2 and V_3 one may use two equilibrium equations of Mechanics and one equation of the three displacements (3), written for support 1-2- 3:

$$\begin{aligned} &\sum F_{zs} = V_1 + V_2 + V_3 \\ &\sum M_{3s} = V_1(b_2 + b_3) + V_2 b_3 \\ &EI\left[w_1 b_3 - w_2(b_2 + b_3) + w_3 b_2\right] = \\ &= \Psi_1 b_3 - \Psi_2(b_2 + b_3) + \Psi_3 b_2 \end{aligned} \tag{5}$$

where:

$\sum F_{zs}$ is the external force after the direction of the axis Oz ;

$\sum M_{3s}$ - the moments and forces couple to the Oy axis through support right (support 3);

Ψ_1, Ψ_2, Ψ_3 - loading functions corresponding to section No.1, 2, 3.

The loading function may be expressed as a sum of the functions of loading appropriate tasks outside Ψ_{1s}, Ψ_{2s}, Ψ_{3s} and the appropriate unknown reaction V_1 , V_2 and V_3:

$$\begin{aligned} &\Psi_1 = \Psi_{1s}\ ; \quad \Psi_2 = \Psi_{2s} - \frac{V_1 b_2^3}{6}\ ; \\ &\Psi_3 = \Psi_{3s} - \frac{V_1(b_2 + b_3)^3}{6} - \frac{V_2 b_3^3}{6} \end{aligned} \tag{6}$$

Note: The loading function is calculated only for outside tasks located on the left of the section in question.

Solving the equation (5) is performed as follows. Taking into account that the displacements of supports 1, 2, 3 have different values depending on the sinks and the functions of loading 3 defined by relations (6), the last of the equation (5) Ψ 2 Ψ1, the results are obtained:

$$\begin{aligned} &\Psi_{1s} b_3 - \left(\Psi_{2s} - \frac{V_1 b_2^3}{6}\right)(b_2 + b_3) + \\ &+\left(\Psi_{3s} - \frac{V_1(b_2 + b_3)^3}{6} - \frac{V_2 b_3^3}{6}\right) b_2 = \\ &= EI\left[w_1 b_3 - w_2(b_2 + b_3) + w_3 b_2\right] \end{aligned} \tag{7}$$

which, with notation:

$$A_{2s} = \Psi_{1s} b_3 - \Psi_{2s}(b_2 + b_3) + \Psi_{3s} b_2 \tag{8}$$

$$B = EI\left[w_1 b_3 - w_2(b_2 + b_3) + w_3 b_2\right] \tag{9}$$

can be made in the form of:

$$\begin{aligned} &\frac{V_1 b_2^3(b_2 + b_3)}{6} - \frac{V_1(b_2 + b_3)^3 b_2}{6} - \\ &-\frac{V_2 b_2 b_3^3}{6} = B - A_{2s} \end{aligned} \tag{10}$$

Multiplying the second of the equation (5) by $\left(b_2 b_3^2\right)/6$ and adding it with equation (10), we obtain the expression:

$$\begin{aligned} &\frac{V_1 b_2 (b_2 + b_3)}{6}\left[b_2^2 - (b_2 + b_3)^2 + b_3^2\right] = \\ &= \frac{b_2 b_3^2}{6} \sum M_{3S} - A_{3S} + B \end{aligned} \tag{11}$$

Resolving the system of the first two equations (5) and the expression (11) we determine the three reaction expressions:

$$\begin{aligned} &V_1 = \frac{1}{b_2(b_2 + b_3)}\left[(A_{2s} - B)\frac{3}{b_2 b_3} - \frac{b_3}{2}\sum M_{3s}\right]\ ; \\ &V_2 = \frac{1}{b_3}\sum M_{3s} - \left(1 + \frac{b_2}{b_3}\right) V_1\ ; \\ &V_3 = \sum F_{zs} - V_1 - V_2\ . \end{aligned} \tag{12}$$

Size B (the relationship (9)) of the expression reaction V1 is determined taking into account both the oscillations of level w_1, w_2 and w_3, and their sign. The displacements w_1, w_2 and w_3 have the plus sign (+) if the oscillations are in a positive sense of the axis after the beam (down) and the minus sign (-), meaning that the oscillations have negative z axis (up). If all supports are at the same level $w_2 = w_1 = w_3 = 0$, then the size defined by the relationship (9) are B = 0 and the reaction expressions (relations (12)) are:

$$V_1 = \frac{1}{b_2(b_2 + b_3)}\left(\frac{3A_{2s}}{b_2 b_3} - \frac{b_3}{2}\sum M_{3s}\right) \tag{13}$$

4. Example of calculation of reaction for the even supports

In order to better understand how the three-displacement equation method works (using the *Ψ* loading function) we present a calculus example used to determine the reaction of rotary kiln supports for clinker portland cement size □ *4,4* x *70 m.*

The scheme to load the rotary kiln is shown in Figure 4. The tasks of this scheme are:

a. Focused tasks:

- The total weight of a rolling ring mounted, which includes weight, the weight of virolite strengthening, the weight of the saboteurs and that of the pawls etc:

$$P_1 = P_2 = P_3 = 783{,}00 \ kN$$

- The weight of the mounted gearwheel, which includes its own weight, the strengthening virolite weight, the weight of the fixing items etc

$$P_5 = 362{,}675 \ kN$$

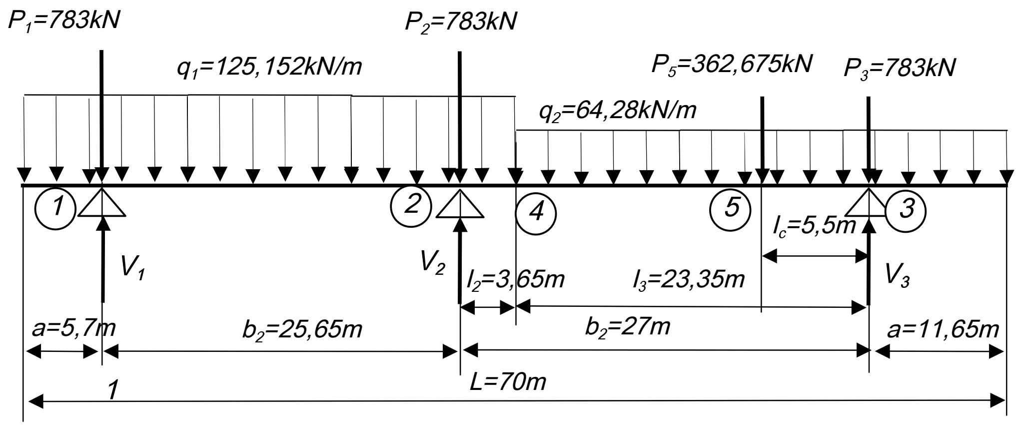

Fig. 4. The scheme to load the rotary kiln

. Uniformly distributed loads (weights due to metal tube, the refractory construction material and subjected to processing in the rotary kiln):

- $q_1 = 125{,}152 \ kN/m$ for the sintering;
- $q_2 = 64{,}280 \ kN/m$ for the burning area.

Considering that the supports are placed at the same level, using relations (5), it is determined:

- The exterior forces after Oz axis direction (see fig. 4):

$$\sum F_{sz} = P_1 + P_2 + P_3 + P_4 + q_1 \cdot L_s + q_2 \cdot L_c =$$
$$= 9341{,}726 \ kN.$$

- The amount of time external forces in relation to an axis parallel to Oy through no. 3 right support (see fig. 4):

$$\sum M_{3s} = P_1 \cdot (b_2 + b_3) + P_2 \cdot b_3 + P_3 \cdot l_c +$$
$$+ q_1 \cdot L_s \cdot (L_s/2 + l_3) + q_2 \cdot l_3^2/2 - q_2 \cdot c^2/2 =$$
$$= 256457{,}965 \ kN \cdot m.$$

- The loading functions of the tasks outside the appropriate support 1, 2, 3 (see relations (4 '), (4) and fig.4):

$$\Psi_{1s} = \frac{q_1 \cdot a^4}{24} = 5504{,}602 \ kN \cdot m^3$$

$$\Psi_{2s} = \frac{q_1 \cdot (a + b_2)^4}{24} + \frac{P_1 \cdot b_2^3}{6} = 7239335{,}90 \ kN \cdot m^3$$

$$\Psi_{3s} = \frac{P_1 \cdot (b_2 + b_3)^3}{6} + \frac{P_2 \cdot b_3^3}{6} + \frac{P_3 \cdot l_c^3}{6} +$$
$$+ \frac{q_1 \cdot \left[(a + b_2 + b_3)^4 - l_3^3\right]}{24} + \frac{q_3 \cdot l_3^4}{24} = 81319921{,}56 \ kN \cdot m^3$$

Using the expression (8), the following values result for A_{2s} :

$$A_{2s} = \Psi_{1s} \cdot b_3 - \Psi_{2s} \cdot (b_2 + b_3) + \Psi_{3s} \cdot b_2 =$$
$$= 1704853580 \ kN \cdot m^2$$

The reactions from supports are determined using the relations (13):

$$V_1 = \frac{1}{b_2 \cdot (b_2 + b_3)} \left(\frac{3 \cdot A_{2s}}{b_2 \cdot b_3} - \frac{b_3}{2} \cdot \sum \overleftarrow{M}_{3s} \right)$$

$$V_1 = 2904{,}858 \ kN;$$

$$V_2 = \frac{1}{b_3} \cdot \sum M_{3s} - \left(1 + \frac{b_2}{b_3}\right) \cdot V_1$$

$$V_2 = 3833{,}970 \ kN;$$

$$V_3 = \sum F_{zs} - V_1 - V_2$$

$$V_3 = 2602{,}898 \ kN$$

5. Example of calculation of reaction for the uneven supports

The moment of inertia of the cross section of the rotary kiln is:

$$I = \frac{\pi}{64} \cdot \left(D_{e,35}^4 - D^4\right) = 1{,}199 \ m^4$$

The longitudinal elasticity modulus of the material of the rotary kiln is:

$$E = 2{,}1 \cdot 10^8 \ kN/m^2$$

One may consider three cases:

a. It is considered the uneven support no. 1, as compared to supports no. 2 and no. 3 with quantities:

$w_1 = 0.005\ m$.
Size *B* (using the relationship (10)) has the value:

$$B = E \cdot I \cdot w_1 \cdot b_3 = 2{,}1 \cdot 10^8 \cdot 1{,}199 \cdot 0{,}005 \cdot 27{,}0$$

$$B = 0{,}34 \cdot 10^8\ kN \cdot m^4$$

Using the relations (9) we obtain the reaction values:

$$V_1 = \frac{1}{b_2(b_2 + b_3)}\left[(A_{2s} - B)\frac{3}{b_2 b_3} - \frac{b_3}{2}\sum M_{3s}\right]$$

$$V_1 = 2795{,}318\ kN.$$

$$V_2 = \frac{1}{b_3}\sum M_{3s} - \left(1 + \frac{b_2}{b_3}\right) \cdot V_1$$

$$V_2 = 4047{,}573\ kN;$$

$$V_3 = \sum F_{zs} - V_1 - V_2$$

$$V_3 = 2498{,}835\ kN$$

b. It is considered the uneven support no. 2, as compared to supports no. 2 and no. 3 with quantities: $w_2 = 0.005\ m$.

Size *B* (using the relationship (10)) has the value:

$$B = -E \cdot I \cdot w_2 \cdot (b_2 + b_3) = -0{,}6628 \cdot 10^8\ kN \cdot m^4$$

Using the relations (9) we obtain the reaction values:

$$V_1 = \frac{1}{b_2(b_2 + b_3)}\left[(A_{2s} - B)\frac{3}{b_2 b_3} - \frac{b_3}{2}\sum M_{3s}\right] =$$

$$V_1 = 3117{,}423\ kN.$$

$$V_2 = \frac{1}{b_3}\sum M_{3s} - \left(1 + \frac{b_2}{b_3}\right) \cdot V_1$$

$$V_2 = 3419{,}468\ kN;$$

$$V_3 = \sum F_{zs} - V_1 - V_2$$

$$V_3 = 2804{,}835\ kN$$

The lowering of the support in the middle leads to the decrease of its own reaction and the increase of reaction to the joined supports.

c. If all three supports are uneven to the reference line:

- support 1 raised by $w_1 = -0{,}002\ m$
- support 2 raised by $w_2 = -0{,}005\ m$
- support 3 lowered by $w_3 = 0{,}004\ m$

Size *B* (using the relationship (10)) has the value:

$$B = EI\left[w_1 b_3 - w_2(b_2 + b_3) + w_3 b_2\right] =$$

$$B = 0{,}7852\ kN \cdot m^4.$$

Using the relations (9) we obtain the reaction values:

$$V_1 = \frac{1}{b_2(b_2 + b_3)}\left[(A_{2s} - B)\frac{3}{b_2 b_3} - \frac{b_3}{2}\sum M_{3s}\right]$$

$$V_1 = 3156{,}672\ kN.$$

$$V_2 = \frac{1}{b_3}\sum M_{3s} - \left(1 + \frac{b_2}{b_3}\right) \cdot V_1$$

$$V_2 = 3342{,}932\ kN;$$

$$V_3 = \sum F_{zs} - V_1 - V_2$$

$$V_3 = 2842{,}122\ kN$$

6. **Conclusions**

The three-displacement equation method, using the loading function ψ, allows the calculus of reactions in the rotary kiln supports rotating for three or more groups of supports.

This method shows that, besides the classic method CLAPEYRON, it is much easier to use and has a higher precision, being an analytical method.

This method allows the determination of reactions also in the case of uneven supports either due to errors of execution and assembly of support groups, or because of accidental causes (oscillation of support foundations, for example).

The example of calculation shows that the oscillation of a support foundation leads to its reaction value change and also to an important change of the nearby supports reaction value.

References

[1] **V.V. Jinescu**, *Utilaj tehnologic pentru industrii de proces IV,* Editura Tehnică, Bucureşti, 1989.[

[2] **M. Atanasiu, G. Jiga**, *Metode analitice noi în Rezistenţa materialelor, 1. Metoda funcţiei de încărcare,* U.P.B., Bucureşti, 1994.

[3] **C. Marin**., *Rezistenţa materialelor şi elemente de teoria elasticităţii,* Editura Bibliotheca, Târgovişte, 2006.

[4] **C. Marin.,** *Aplicaţii ale teoriei elasticitatii si plasticitatii in inginerie,* Editura Bibliotheca, Târgovişte, 2007.